Torsional Alfvén waves in Jupiter‘s metallic hydrogen region

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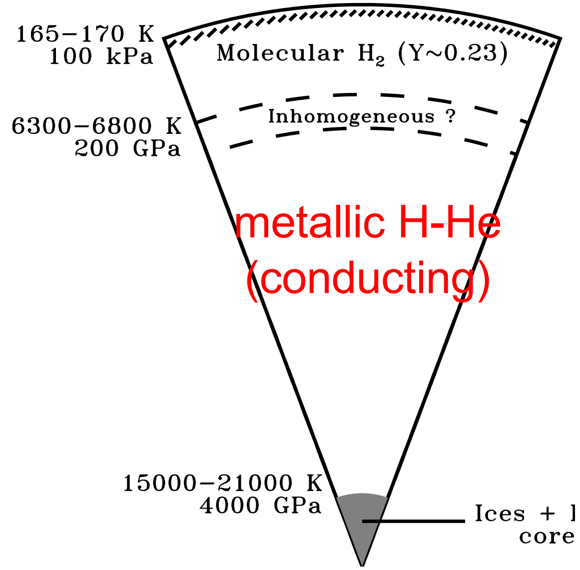
Jupiter’s metallic hydrogen region

- The origin of the magnetic field/dynamo action
  - poorly known in data
- How to infer the deep interior dynamics?
  - through the magnetic field:
    - pre-Juno ($n \lesssim 4$): strong, predominantly axial dipole, secular variation?
    - **post-Juno (**$n \lesssim 10$ **and more?):** closest as ever to a dynamo region: localized patches
  - through any oscillations/waves?
    - an electrically-conducting, low-viscous fluid in a rapidly-rotating spherical shell permeated by the magnetic field
      - Lorentz/Coriolis = O(1)?
      - the rotating MHD hosts a variety of waves

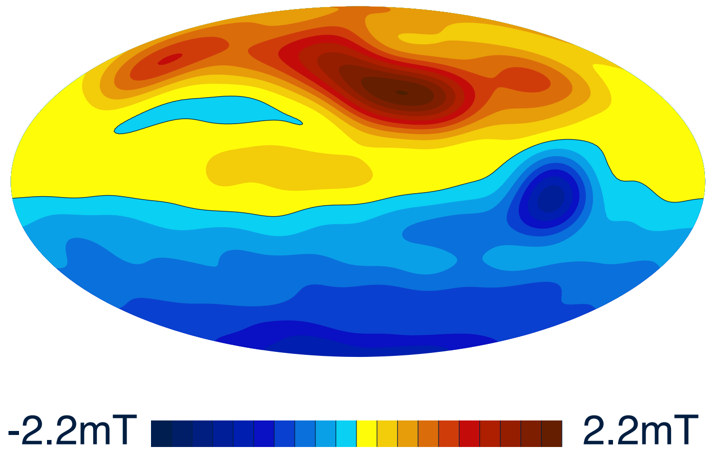
Predicted internal structure

(Guillot 1999)



Br inferred at surface 0.96 R_J

(JRM09: Connerney et al. 2018)



Rotating MHD waves

- Waves in the presence of both magnetic field and rotation have been studied for **incompressible fluids** and applied to **Earth's liquid iron core**
  - **torsional Alfvén waves** (e.g. Braginsky 1967, Zatman & Bloxham 1997)
    - e.g. ~ 6 yrs variation → core internal field Bs >~ 2 mT (Gillet et al. 2010)
    - accounting for the interannual length-of-the-day variations?
  - magnetic Rossby waves (Hide 1966)
    - e.g. ~ 300 yrs westward drift → B_ϕ ~ 1-10 mT? (Hori et al. 2015)
  - MAC waves in a thin stably-stratified layer, at the top of the core?
    - axisymmetric (e.g. Braginsky 1993; Buffett 2014), fast magnetic Rossby (Chulliat et al. 2015)
- What about in Jupiter's interior?
  - density significantly varies with radius: $\rho(r_{core})/\rho(r_{metallic})$ ~< 20

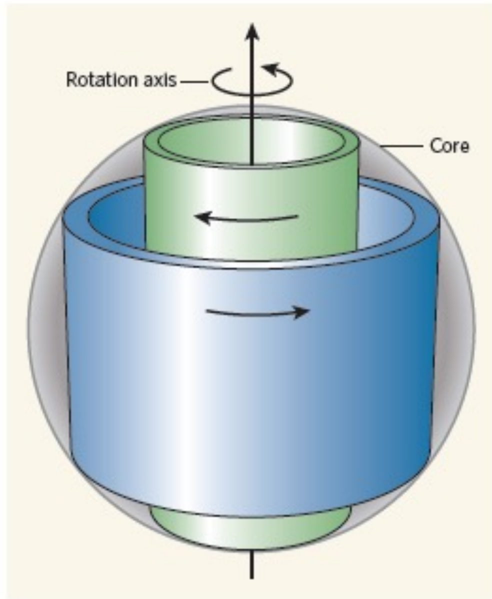
    * anelastic approximation for compressible fluids adopted

Torsional Alfvén waves

- A special class of Alfvén waves (Braginsky 1970; also Jault & Finlay 2015) :
  - The azimuthal momentum equation integrated over cylindrical surfaces C = 2πs h(s) about the rotation axis:

$$\frac{\partial}{\partial t}\int_{\mathcal{C}} \overline{\rho} u_\phi dS + \int_{\mathcal{C}} \hat{\boldsymbol{e}}_\phi \cdot (\nabla \cdot \overline{\rho}\boldsymbol{u}\boldsymbol{u}) dS + 2\Omega \int_{\mathcal{C}} \overline{\rho} u_s dS = \int_{\mathcal{C}} \hat{\boldsymbol{e}}_\phi \cdot (\boldsymbol{J} \times \boldsymbol{B}) dS$$

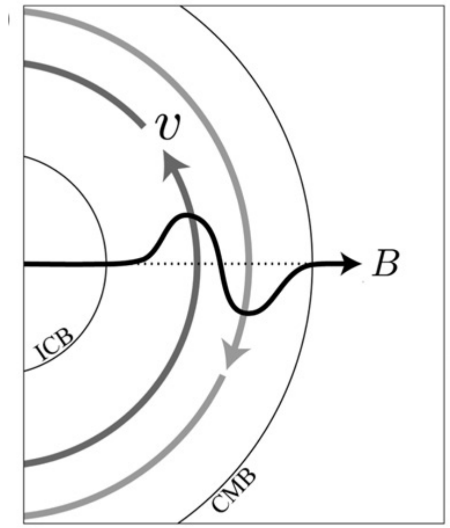
  - For anelastic/incompressible fluids, the Coriolis term vanishes
  - The magnetostrophic balance (Ro, E<<1 & Λ=O(1)) yields a steady state (Taylor 1963)



  - Cylindrical perturbations on the state, $\langle \overline{u}_\phi' \rangle = \langle \overline{u}_\phi' \rangle(s,t)$, can be governed by a homogeneous equation:

$$\frac{\partial^2}{\partial t^2}\frac{\langle \overline{u'_\phi} \rangle}{s} = \frac{1}{s^3 h \langle \overline{\rho} \rangle}\frac{\partial}{\partial s}\left(s^3 h \langle \overline{\rho} \rangle U_\mathrm{A}^2 \frac{\partial}{\partial s}\frac{\langle \overline{u'_\phi} \rangle}{s} \right)$$

    - propagation in radius s with Alfvén speed U_A given by z-mean quantities: $U_A = (\langle \overline{B}_s^2 \rangle / \langle \overline{\rho} \rangle \mu_0)^{1/2}$
    - both outward (+s) and inward (-s) propagation, or standing waves, possible

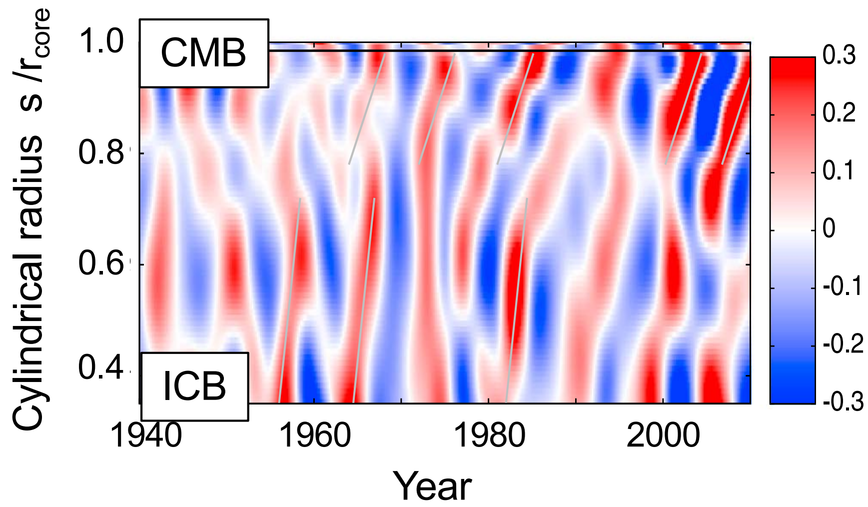


(Roberts & Aurnou 2012)

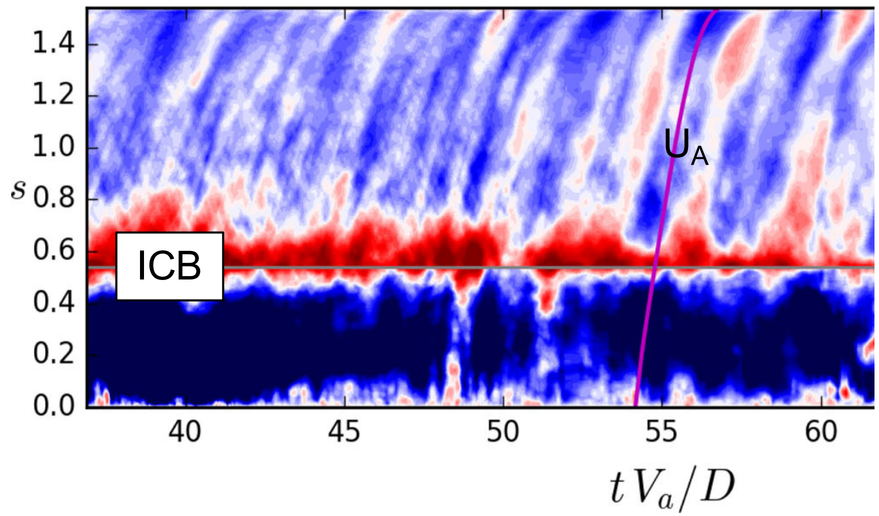
Torsional waves in Earth's core

- Suppose the incompressible case
  - Alfvén speed U_A for constant ρ
- Early studies sought its standing form
  (e.g. Braginsky 1970; Zatman & Bloxham 1997)
- More likely travelling to the equator
  - data: 4-9 year periods (Gillet et al. 2010, 2015)
    - the internal field strength of $\langle B_s^2 \rangle^{1/2} \geq 2$ mT
  - geodynamo simulation (Wicht & Christensen 2010; Teed et al. 2014; Schaeffer et al. 2017)
    - **no obvious reflection, no standing 'oscillations'**
    - due to strong dissipation around CMB?
  - lab experiments also? (Nataf et al.)

Axisymmetric zonal velocity u_ϕ
in a core flow model (Gillet et al. 2015)



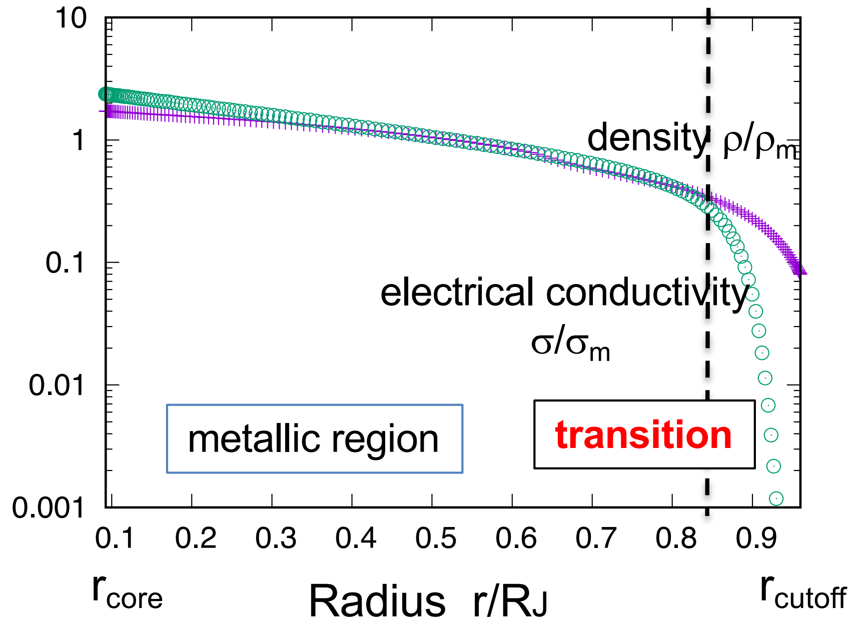
in a dynamo simulation (Schaeffer et al. 2017)



Jovian dynamo models

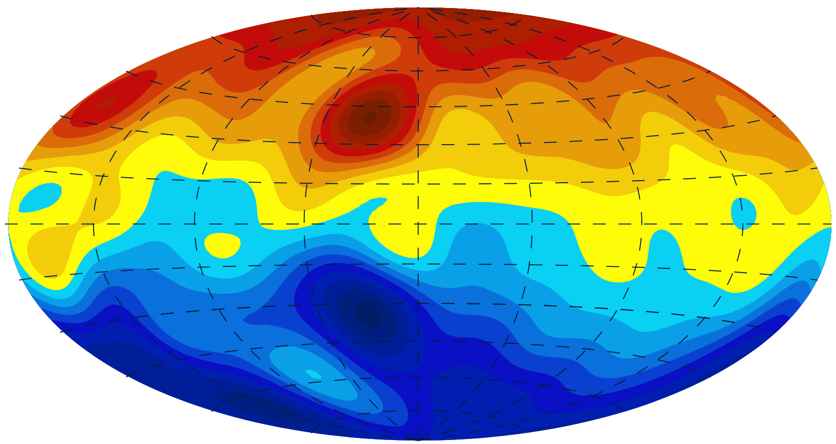
- Setup (Jones 2014; also Gastine et al. 2014):
  - model a metallic region & a transition to the molecular region: $0.09R_J \lesssim r \lesssim 0.96R_J$
  - dynamos driven by rotating, anelastic convection (Lantz & Fan 1999; Braginsky & Roberts 1995)
  - a reference state (French et al. 2012):
    - **density contrast**, $\rho(r_{core})/\rho(r_{cutoff}) \sim 18$
    - electrical conductivity σ drops at $r \sim 0.85R_J$ by more than five orders

The reference state used in the model



- Some features:
  - jupiter-like magnetic fields reproduced

Br at the cutoff radius $r_{cutoff} \sim 0.96\ R_J$ truncated up to n=10 (after Jones 2014)



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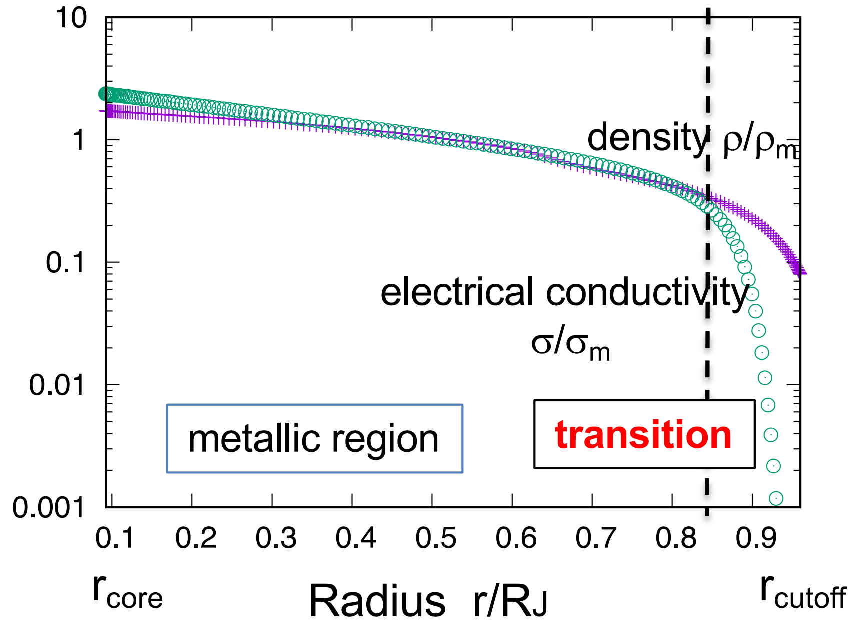
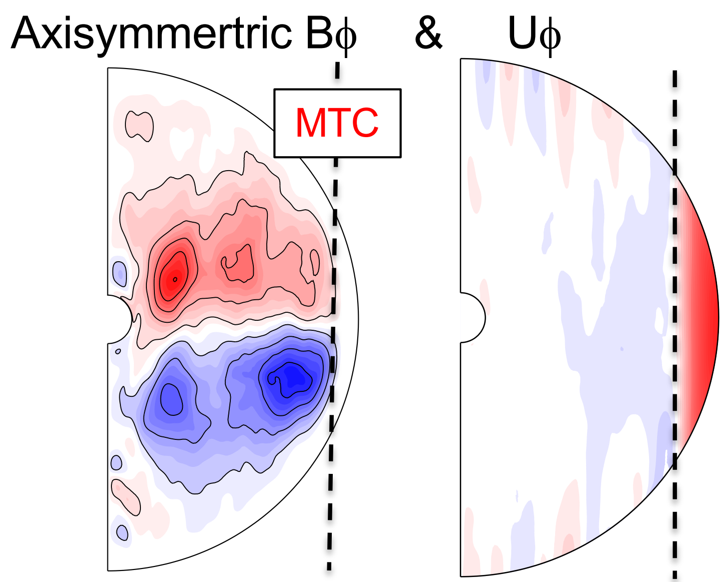
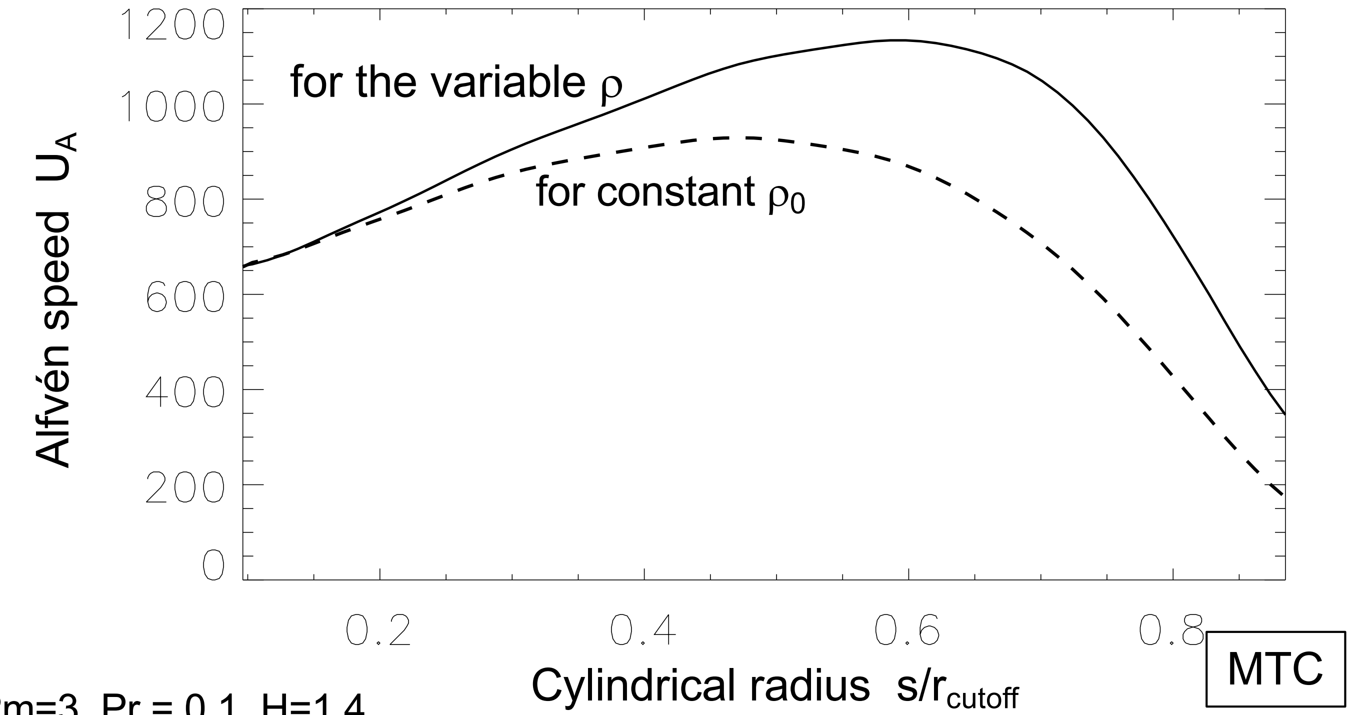


- Some features:
  - jupiter-like magnetic fields reproduced
  - a magnetic tangent cylinder formed
    - attaching to a top of the metallic region at the equator
    - one strong jet outside the MTC; weak multiple zonal flows inside
      - fluctuating: to be analyzed



Anelastic Alfvén speed in simulations

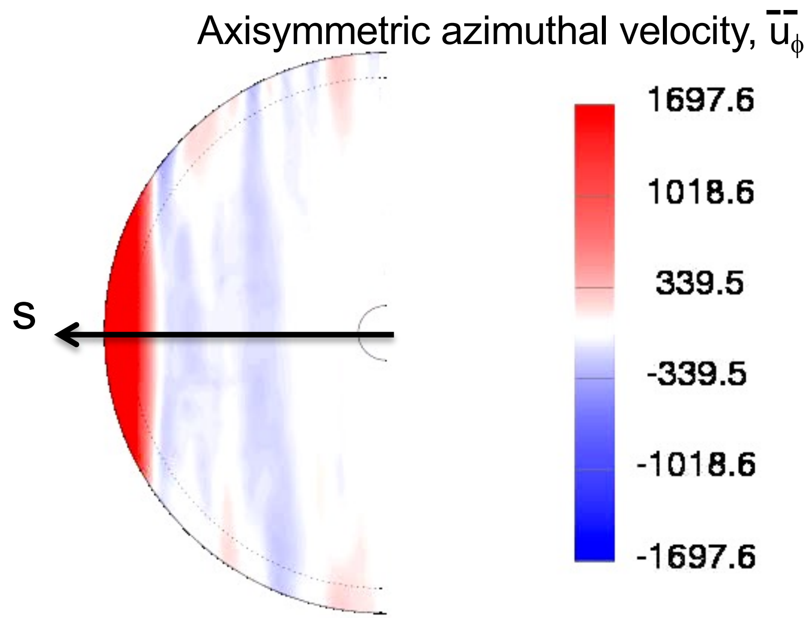
- Predicted Alfvén speeds
  $U_A = (\langle \overline{\overline{B_s}}^2 \rangle / \mu_0 \langle \overline{\overline{\rho}} \rangle)^{1/2}$:
  - independent of wavenumbers, i.e. nondispersive
  - higher for low ρ, i.e. increasing with s
  - drops to the MTC

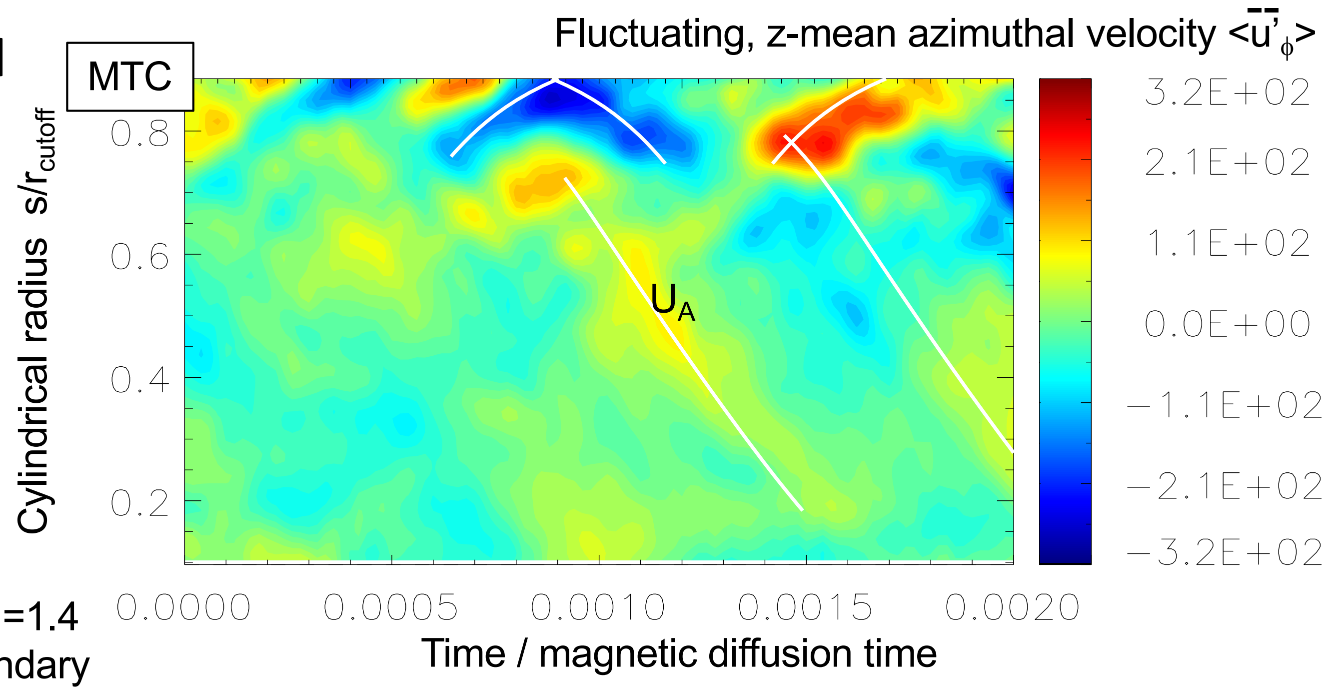


at E = $1.5*10^{-5}$, Pm=3, Pr = 0.1, H=1.4
& fixed entropy-flux outer boundary

Torsional waves in Jovian simulations

- Identified with the predicted speeds of $U_A = (\langle\overline{B}_s^2\rangle/\mu_0\langle\overline{\rho}\rangle)^{1/2}$
  - travelling in s, outwardly or inwardly, from an outer radius ($0.6 < s/r_{cutoff} < 0.8$)
- Reflected from the MTC
  - which acts as an interface to a resistive zone
    - 1D models helpful



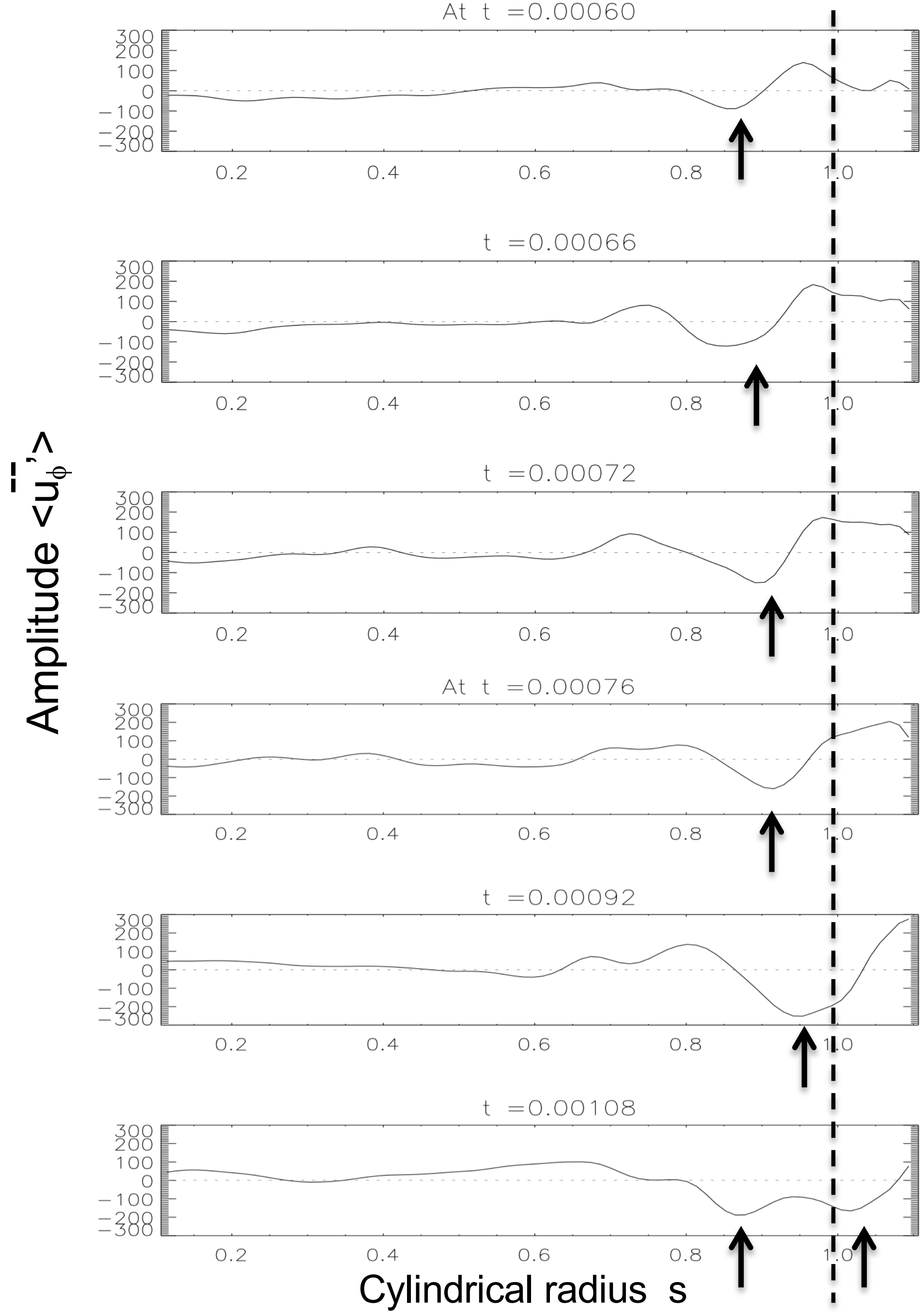


at $E = 1.5*10^{-5}$, Pm=3, Pr = 0.1, H=1.4 & fixed entropy-flux outer boundary

Evolution of torsional waves

- Waveforms can become sharp
  - steepening; weak, unstable
    - typical for inviscid nonlinear waves
    - e.g. water waves, shock waves
    - cf. dispersive, cnoidal/solitaty Rossby ones (Hori et al. 2017)

- Reflection from the MTC
  - as well as transmission to the outside
  - reflected waves not identical to incident waves
    - due to its spherical geometries, variable background fields, nonlinearities, etc.



Alfvén waves approaching a resistive layer

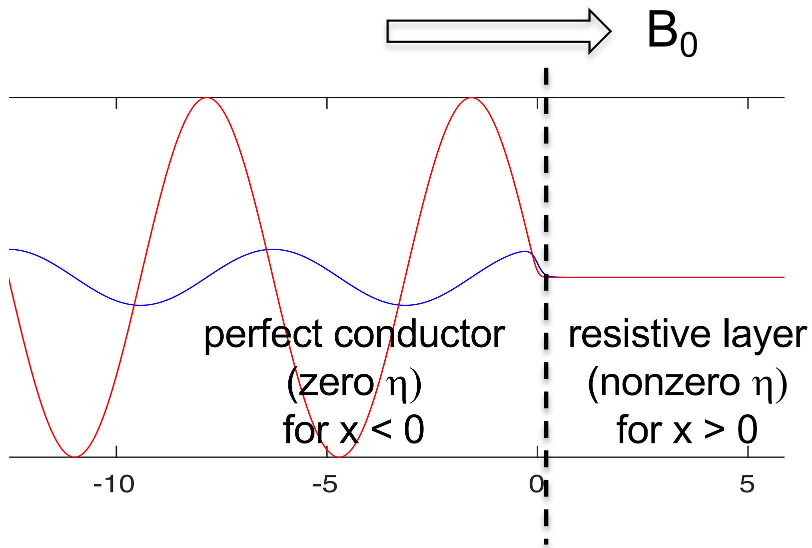
- Consider 1d models:

$$\boldsymbol{B} = B_0 \hat{\boldsymbol{e}}_x + b_y(x)\hat{\boldsymbol{e}}_y \ , \quad \boldsymbol{u} = u_y(x)\hat{\boldsymbol{e}}_y$$

then the governing equations

$$\frac{\partial b_y}{\partial t} = B_0 \frac{\partial u_y}{\partial x} + \frac{\partial}{\partial x}\eta\frac{\partial b_y}{\partial x} \qquad \frac{\partial u_y}{\partial t} = \frac{B_0}{\mu\rho}\frac{\partial b_y}{\partial x}$$

where B_0 and η are constants; $\eta = 0$ for $x < 0$



- Seek solutions in form of

$$b_y = e^{i\omega t}\left(e^{-ikx} + \mathcal{R}\, e^{+ikx}\right) \qquad \text{for } x < 0$$

$$b_y = \mathcal{T}\, e^{i\omega t} e^{\lambda x} \qquad \text{for } x > 0 \ \text{(with complex } \lambda\text{)}$$

with continuous conditions across the interface x = 0:

$$b_y^- = b_y^+ \ , \quad \frac{\partial b_y^-}{\partial x} = \frac{\partial b_y^+}{\partial x}$$

to yield the reflection coefficients for $\omega >> V_A^2/\eta_0$:

$$\mathcal{R} = \frac{ik + \sqrt{\omega/2\eta_0}(-1+i)}{ik - \sqrt{\omega/2\eta_0}(-1+i)} \ , \quad \mathcal{T} = 1 + \mathcal{R}$$

  - for large $\omega >> k^2\eta_0$, then R ~ -1 & T ~ 0: perfect reflection
  - $u_y \sim db_y/dx$: a negative reflection in b_y yields a positive reflection in u_y

Excitation mechanism

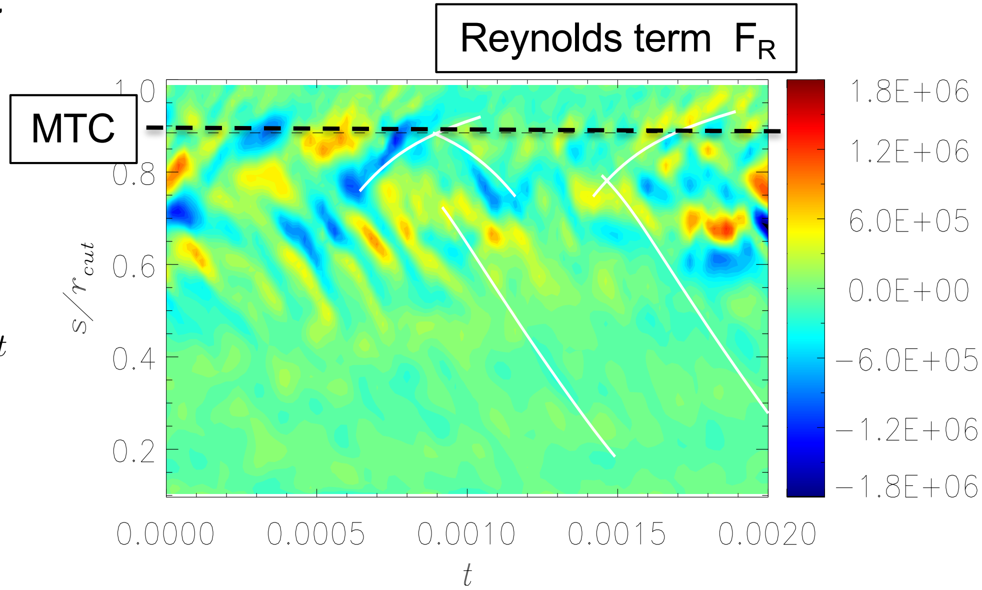
- The momentum equation can be split into the restoring and forcing parts:

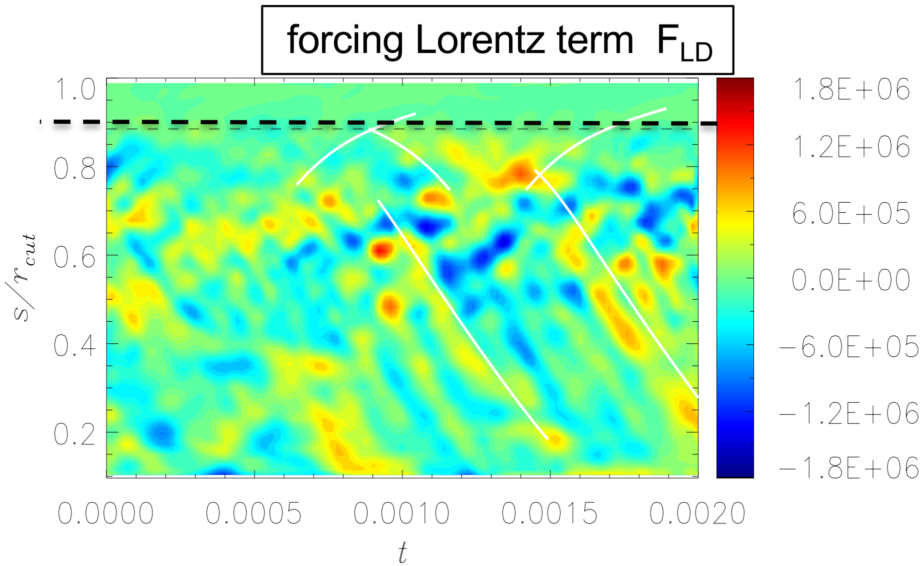
$$F_{\rm R} = -\frac{1}{s^2 h}\frac{\partial}{\partial s} s^2 h \,\langle \overline{\rho}\ \overline{u_s u_\phi}\rangle$$

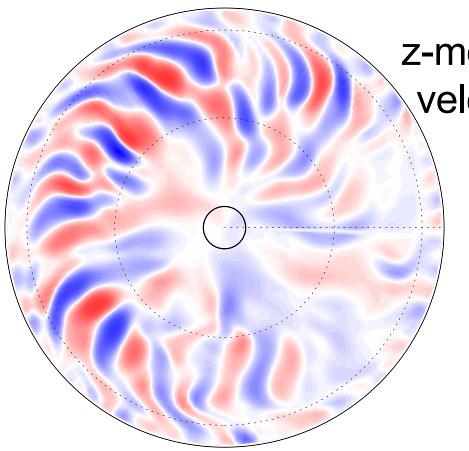
$$F_{\rm LD} = F_{\rm L} - F_{\rm LR}$$

$$= \frac{1}{\mu_0 s^2 h}\frac{\partial}{\partial s} s^2 h \,\langle \overline{B_s B_\phi}\rangle - \int^{\tau}\left[\frac{1}{s^2 h}\frac{\partial}{\partial s}\left(s^3 h \langle\overline{\rho}\rangle U_{\rm A}^2 \frac{\partial}{\partial s}\frac{\langle\overline{u'_\phi}\rangle}{s}\right)\right] dt$$

  - TW initiated by the Reynolds force at an outer radius, $0.6 < s/r_{cutoff} < 0.8$
  - at which convection is beating on timescales of (hydro) Rossby waves





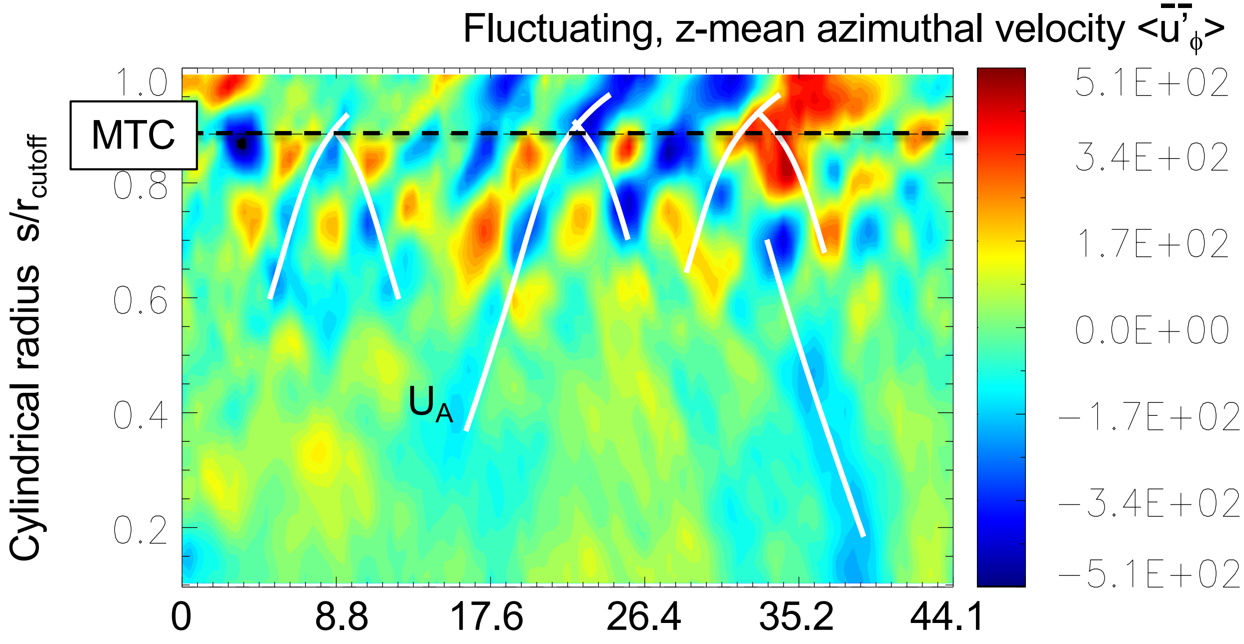


z-mean radial velocity <u_s>

Torsional ‘oscillations’ possible

- Zonal flow fluctuations in another case
  - standing inside the MTC
    - travelling from an outer radius both inwardly and outwardly
    - superposition with reflected waves enables standing waves
  - only transmitted outside the MTC
    - while being absorbed

    → The nature signifying the depth?
  - cf. Earth’s CMB
    - a bound of the core fluid (e.g. Schaeffer & Jault 2016)



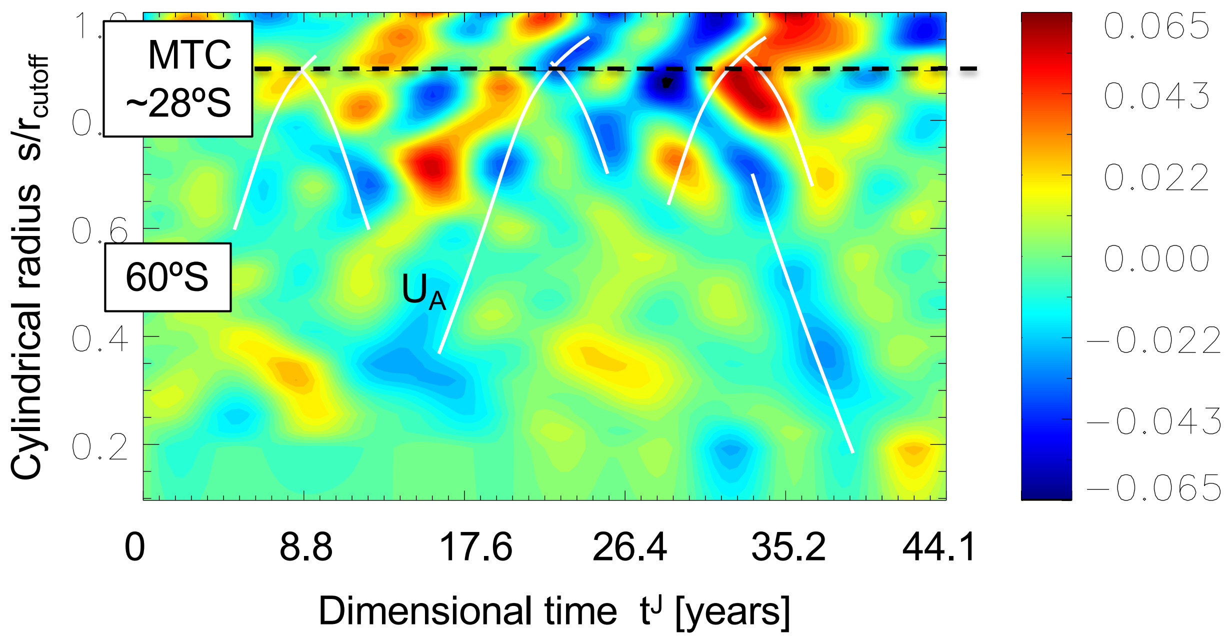
at E = $1.5*10^{-5}$, Pm=3, Pr = 0.1, H=1.4
& fixed entropy outer boundary

Detectable on Jupiter?

- Typical timescales
  - Given a field of **Bs ~ 3 mT** & ρ ~ 853 kg/m^3 at the equator at a top of the metallic region (~ 0.85 R_J), then Alfvén speed ~ 9.2*10^{-2} m/s
  - TW traveltimes across the metallic region can be **9-13 years**
    - Note: the internal field uncertain

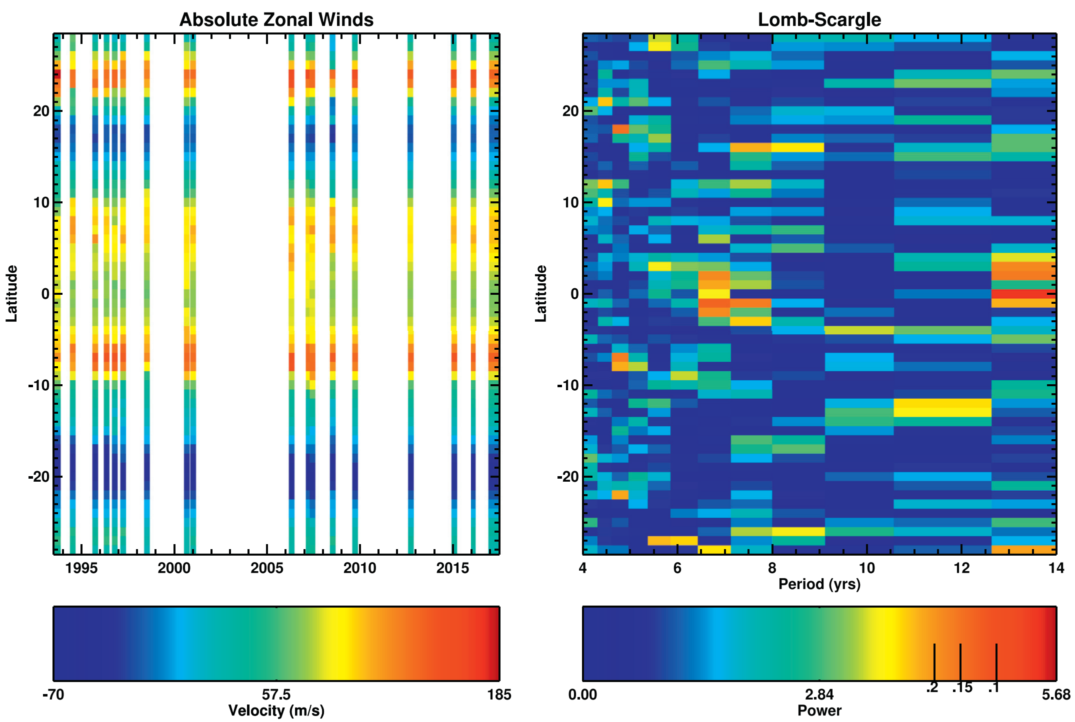
- TW seen on a spherical surface above the metallic region
  - amplitude < 1/10 of our zonal jet outside MTC
  - cf. changes in the zonal wind at the cloud level? (Tollefson et al. 2017)
  - cf. global upheavals?? (e.g. Rogers 1995)

Filtered, zonal velocity fluctuation $\overline{\overline{u'}}_\phi$ /max($\overline{\overline{U}}_\phi$) at the cutoff boundary (~0.96 R_J) in the southern hemisphere

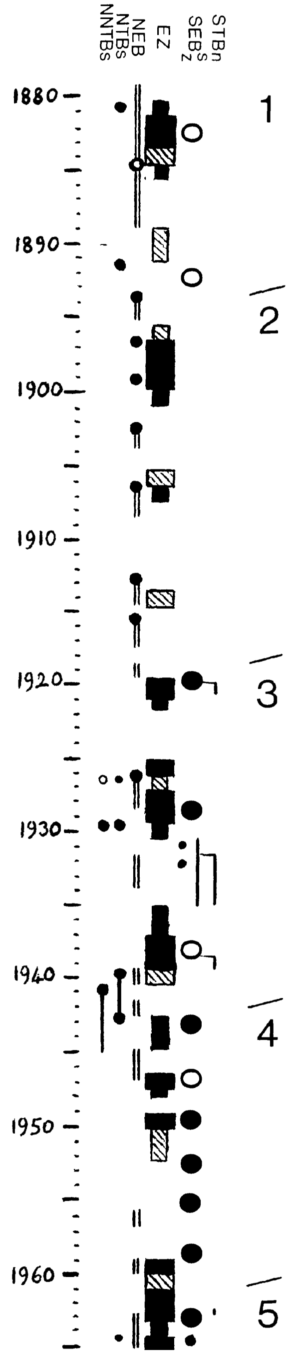


Long-term changes at the cloud deck?

- Zonal wind speed
  - In-situ (Cassini vs. Voyager 2) reported (Porco et al. 2003)
  - ground/HST campaigns (2009-2016) identified relevant variability near 24 °N & 5-7 year periods at lower latitudes (Tollefson et al. 2017)
- Coloration, brightening, outbreak events, etc.
  - sketched for > 100 years: ‘global upheavals’ (Rogers 1995; Fletcher 2017)
  - irregularly, but periodic at some epoch at NTB?



(Tollefson et al. 2017)



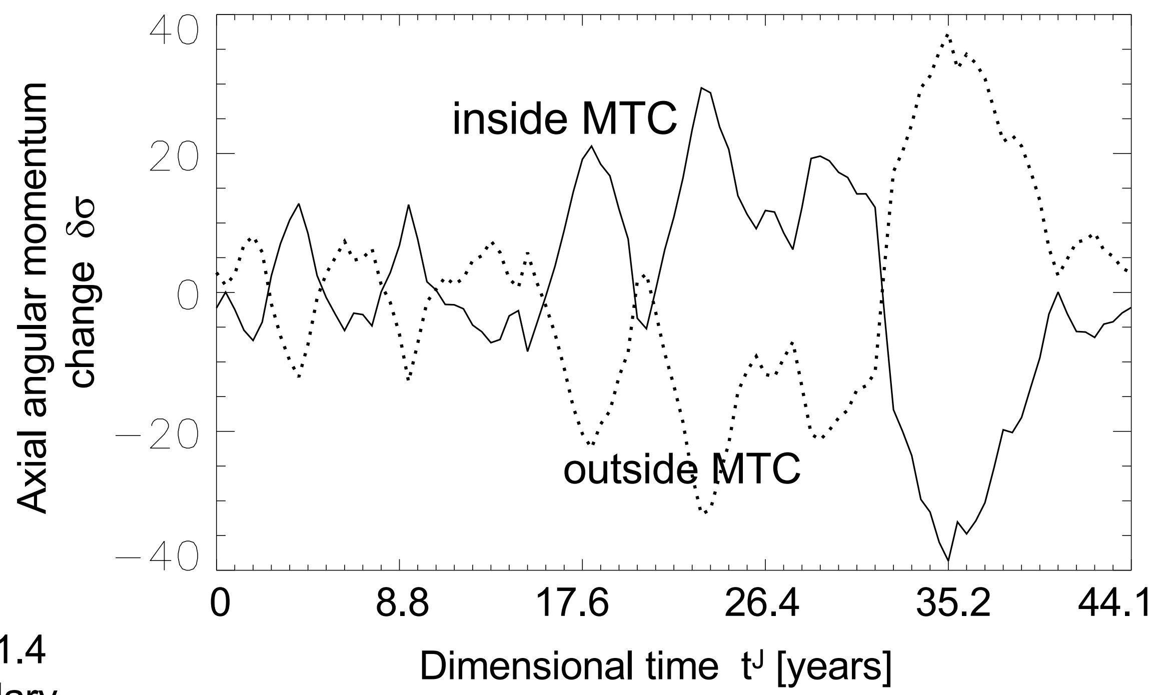
(Rogers 1989, 1995)

Length-of-day variations

- TW transport the angular momentum
  - almost-perfectly exchanging the angular momentum $\delta\sigma$ with the overlying molecular region, where

$$\delta\sigma = 2\pi \int_{s_{\rm tc}}^{s_{\rm mtc}} \int_{z_-}^{z_+} h\langle\rho_{\rm eq}\rangle s^2 \langle\overline{u'_\phi}\rangle dz ds$$

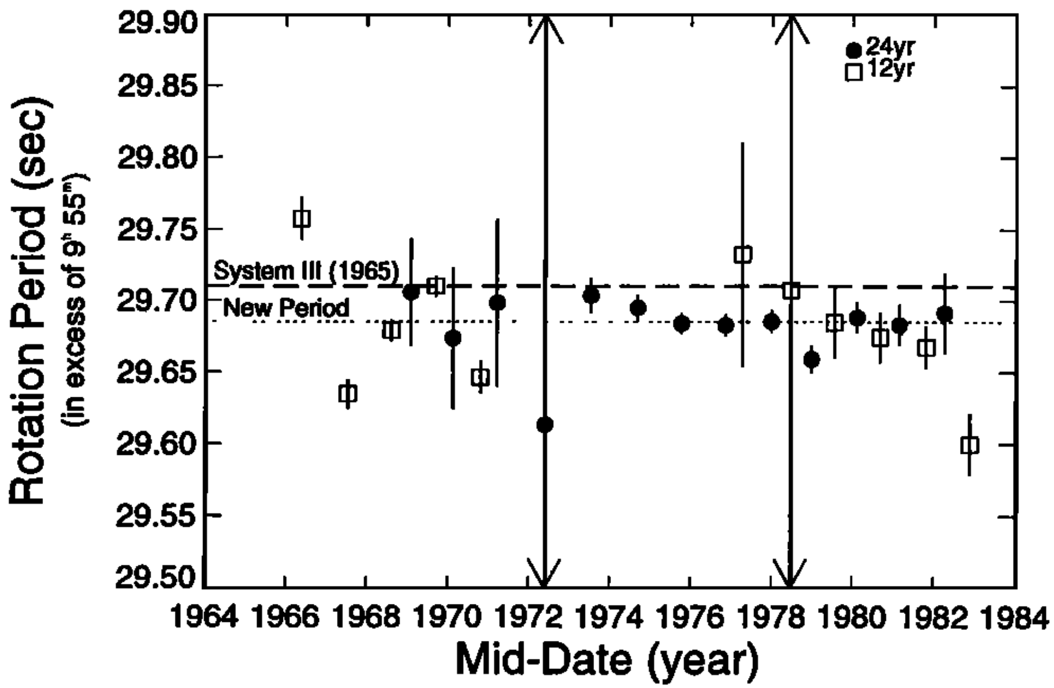
- This may fluctuate the planet's rotation rate (LOD)
  - the change $\delta\sigma = -2\pi I \delta P/P^2$ implying an LOD variation δP
  - cf. O(10^{-2} s) changes of the System III ?: decametric radio emission (Higgins et al. 1996)



at E = $1.5*10^{-5}$, Pm=3, Pr = 0.1, H=1.4
& fixed entropy outer boundary

Jovian LOD changes?

- The gas giant's rotation rate
  - System III (1965): 9h 55m 29.71s
    - relying on measurements of decametric radial emission from the magnetosphere (Burke & Franklin 1955)
    - the accuracy in O(10^{-2}s) has been some debate
      - the true change (Higgins et al. 1996, 1997)
      - jovimagnetic SV (Russell et al. 2001; Ridley & Holme 2016)
  - what else??



(Higgins et al. 1996)

Summary

Axisymmetric, torsional Alfvén waves possibly excited in Jupiter’s metallic H region

- identified in Jovian dynamo simulations
  - implementing a smooth transition from the metallic to molecular regions, forming a magnetic TC

- propagating in cylindrical radius with Alfvén speeds ~ $B_s/\rho^{1/2}$
  - on timescales of O(10^{0-1} yrs) for an equatorial field of 1-3 mT
    - Note: the dimensional values may vary
  - reflections from MTC, also standing ‘oscillations’, may reveal the radius
  - angular momentum exchanges with the overlying molecular region, fluctuating LOD
  - detectable in surface zonal flows beyond the metallic region

Thank you

Anelastic spherical dynamo simulations

- MHD dynamos driven by anelastic convection in rotating spherical shells
  - adopting the Lantz-Braginsky-Roberts formalism (Lantz & Fan 1999; Braginsky & Roberts 1995; also Jones+ 2011)
  - dimensionless, governing equations about the reference state:

$$\nabla \cdot \overline{\rho}\, \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{Pm}{E}\left[\nabla \hat{p} + 2\hat{\boldsymbol{e}}_z \times \boldsymbol{u} - \frac{1}{\overline{\rho}}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}\right] - \frac{Pm^2 Ra}{Pr}\frac{d\overline{T}}{dr} S \hat{\boldsymbol{e}}_r + Pm\, \boldsymbol{F}_{\mathrm{V}}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \nabla \times (\overline{\eta} \nabla \times \boldsymbol{B}) \,, \quad \nabla \cdot \boldsymbol{B} = 0$$

$$\frac{\partial S}{\partial t} + \boldsymbol{u} \cdot \nabla S = \frac{Pm}{Pr}\left[\frac{1}{\overline{\rho}\overline{T}} \nabla \cdot \left(\overline{\rho}\overline{T} \nabla S\right) + H\right] + \frac{Pr}{Pm Ra \overline{T}}\left[\frac{1}{E}\frac{\overline{\eta}}{\overline{\rho}}(\nabla \times \boldsymbol{B})^2 + Q_{\mathrm{V}}\right]$$

  - with Ekman, kinetic/magnetic Prandtl, and Rayleigh numbers with mid-depth values (X_m):

$$E = \frac{\nu}{\Omega d^2} \,, \quad Pr = \frac{\nu}{\kappa} \quad Pm = \frac{\nu}{\eta_{\mathrm{m}}} \,, \quad Ra = \frac{T_{\mathrm{m}} d^2 \Delta S}{\nu \kappa}$$

E	Pr	Pm	Ra
$(1.5\text{-}2.5)*10^{-5}$	0.1	3	$O(10^{7})$
$O(10^{-18})$	0.1-1	$O(10^{-7})$	

- Leeds spherical dynamo code: based on pseudo spectral method