Convection in solids with melting and freezing at either or both boundaries

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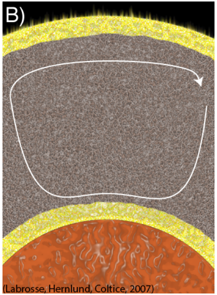
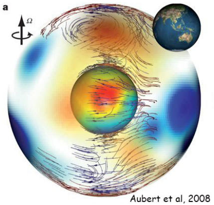
Outline

Solid–liquid interfaces in planetary sciences

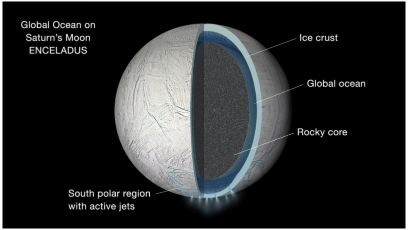


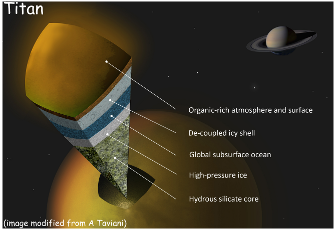
(Labrosse, Hernlund, Coltice, 2007)



Aubert et al, 2008

- Convection in planetary mantles interacting with a liquid layer above and/or below. Applies to:
  - magma ocean above the mantle during its crystallisation ($\sim 10\mathrm{Ma}$).
  - Basal magma ocean for a longer period (few Gyr, Labrosse et al, 2007).
  - Icy satellites with a buried ocean below one or between two possibly convecting ice layers.
  - The inner core of terrestrial planets.





(image modified from A Taviani)

Outline

We consider a solid that behaves like a very viscous fluid on long time-scales $\Rightarrow$ Infinite Prandtl number.

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$-\nabla p + \nabla^2 \boldsymbol{u} + Ra\, T \boldsymbol{e}_z = 0 \tag{2}$$

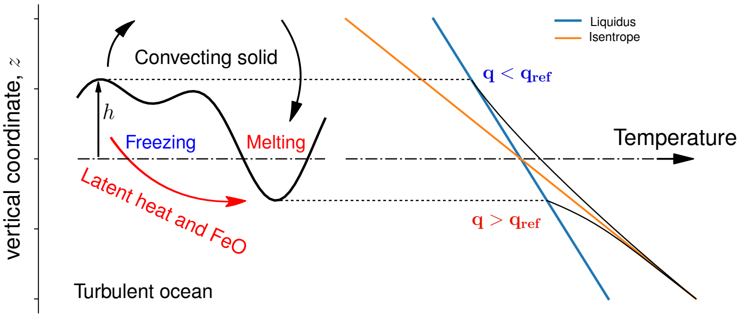
$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \nabla^2 T + H \tag{3}$$

Usual boundary conditions:

- Imposed temperature owing to efficient mixing in adjacent domain (atmosphere, ocean, liquid core).
- Non-penetrative: $u_z = 0$ on a horizontal boundary.
- Free-slip: $\partial_z u_x = \partial_z u_y = 0$.

But in fact, flow in the solid $\Rightarrow$ dynamic topography.

Phase change boundary conditions



- Viscous stress in the solid mantle ⇒ topography builds with timescale $\tau_\eta = \frac{\eta}{\Delta\rho g d}$
- Heat transfer in the liquid erases topography with timescale $\tau_\phi = -\frac{\rho_s L}{\rho_l c_{pl} u_l \frac{\partial T_m}{\partial r}}$
- Competition of the two processes controlled by $\Phi = \frac{\tau_\phi}{\tau_\eta}$

$$-\Phi v_r + 2\frac{\partial v_r}{\partial r} - p = 0$$

- $\Phi \to \infty \Rightarrow$ classical non-penetrative boundary condition ($v_r = 0$).
- $\Phi \to 0 \Rightarrow$ permeable boundary condition ($v_r \neq 0$).

Outline

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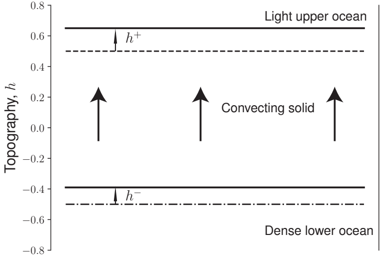
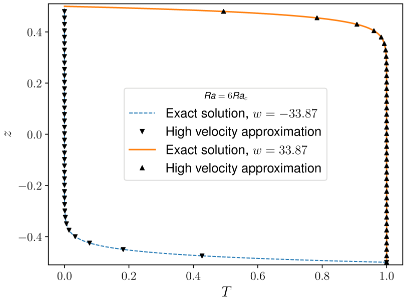
The translation mode of convection

Labrosse et al. (2018)



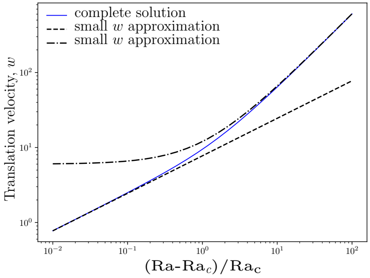


- Rigid vertical translation of the solid with continuous phase change at each boundary is possible if

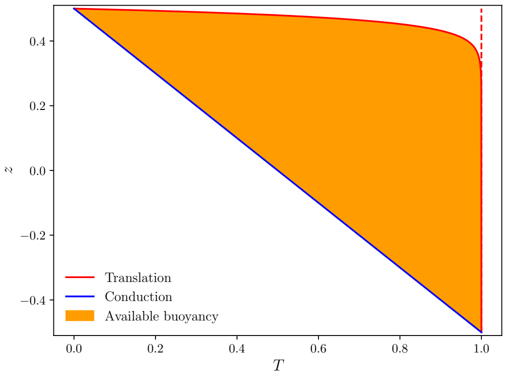
$$Ra \geq Ra_c = 12\left(\Phi^+ + \Phi^-\right),$$

- In the large Rayleigh number limit ($Ra > 2Ra_c$)

$$|u_z| = Nu = \frac{6Ra}{Ra_c}$$



Physical interpretation



- The extra weight of the topography is balanced by the buoyancy associated with the high temperature, i.e. assuming an infinitely thin boundary layer:

$$\alpha\rho_0 g\frac{\Delta TH}{2} = \Delta\rho^+ gh^+ + \Delta\rho^- gh^-,$$

- The topography is related to the velocity by

$$h^{\pm} = \tau_{\phi^{\pm}} u_z.$$

- In dimensionless form:

$$u_z \sim \pm\frac{Ra}{2\left(\Phi^+ + \Phi^-\right)} = \pm\frac{6Ra}{R_c}$$

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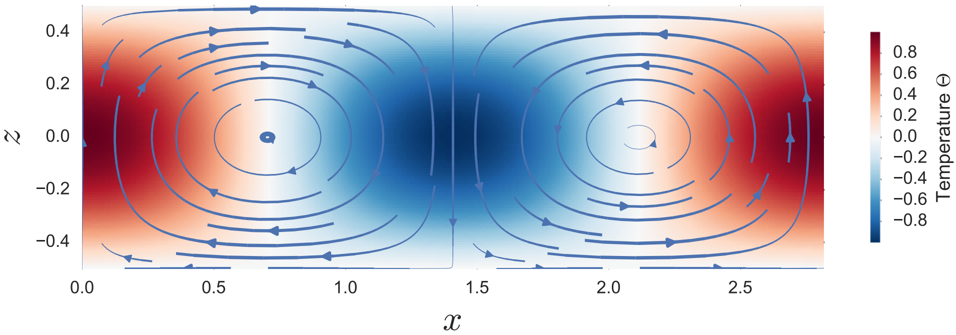
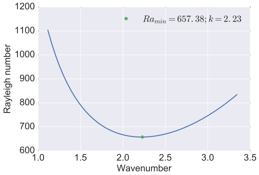
Convective modes at onset for $\Phi^+ = \Phi^- \equiv \Phi^{\pm}$

$\Phi^+ = \Phi^- = 10^5$:



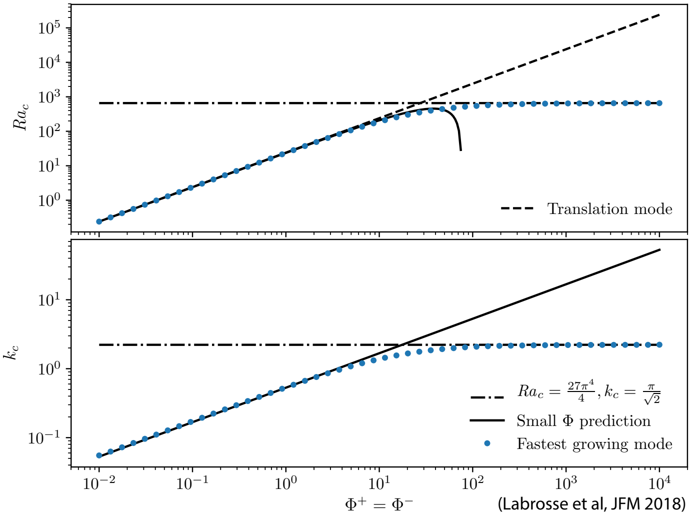


Close to Rayleigh-Bénard value for classical free-slip boundary conditions:

$$Ra_c = \frac{27\pi^4}{4}; \qquad k_c = \frac{\pi}{\sqrt{2}}$$

$\Phi^+ = \Phi^- = 10$:

Onset of convection with $\Phi^+ = \Phi^-$



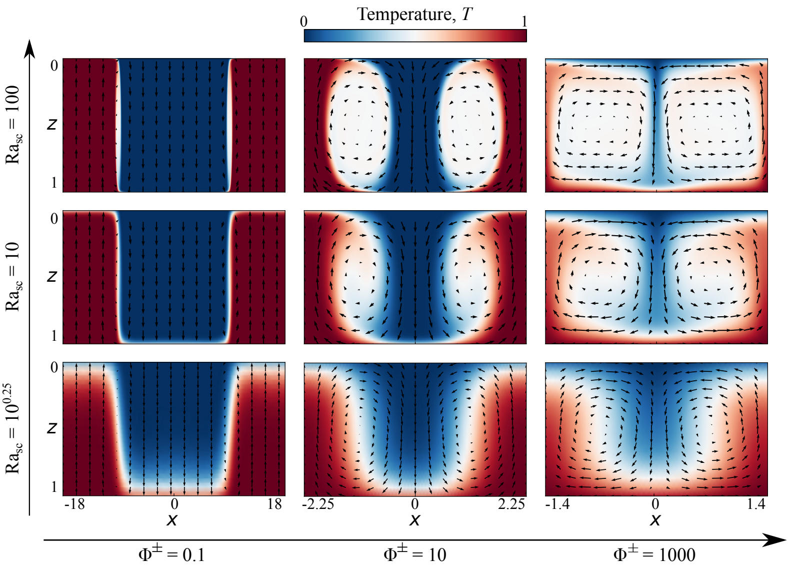
(Labrosse et al, JFM 2018)

At low $\Phi^\pm$, Ra_c gets close to but stays lower than that for pure translation.

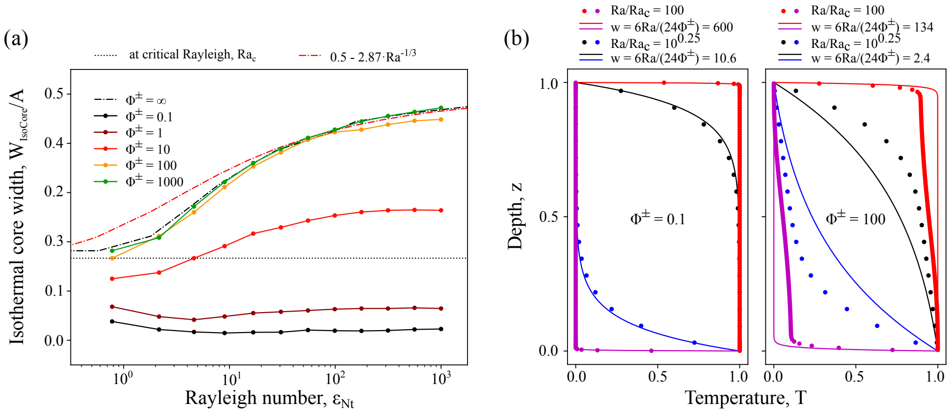
Outline

DNS using mantle convection code StagYY

Agrusta et al. (2019)



Similarity with the translation mode

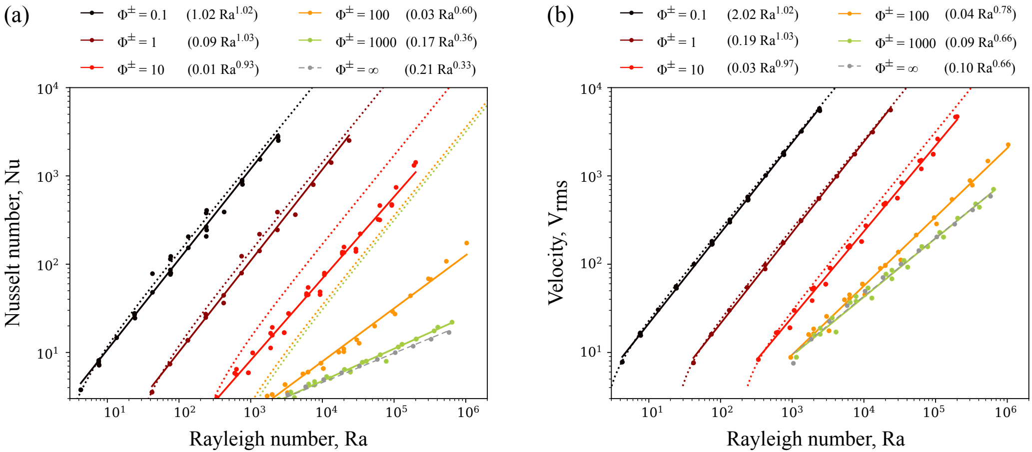


- $\Phi \leq 1$: thermal structure in each vertically moving block similar to that of the translation mode.

Heat transfer and velocity

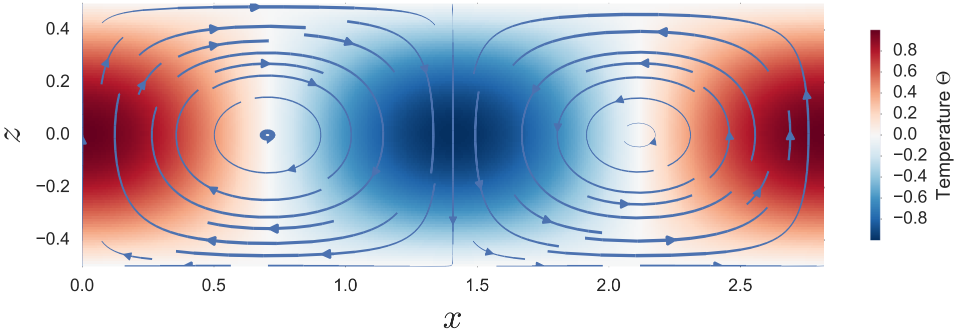
- Dashed lines: weakly non–linear predictions to first order
- Symbols: DNS results
- Solid lines: power law fits.

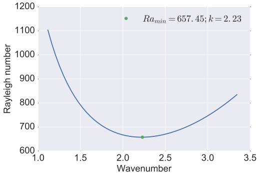
- $\Phi \gg 1$: classical $Nu \sim Ra^{1/3}$
- $\Phi \lesssim 1 \Rightarrow Nu \sim Ra/\Phi$



Convective modes at onset as function of Φ^- ($\Phi^+ = \infty$)

$\Phi^- = 10^5$:





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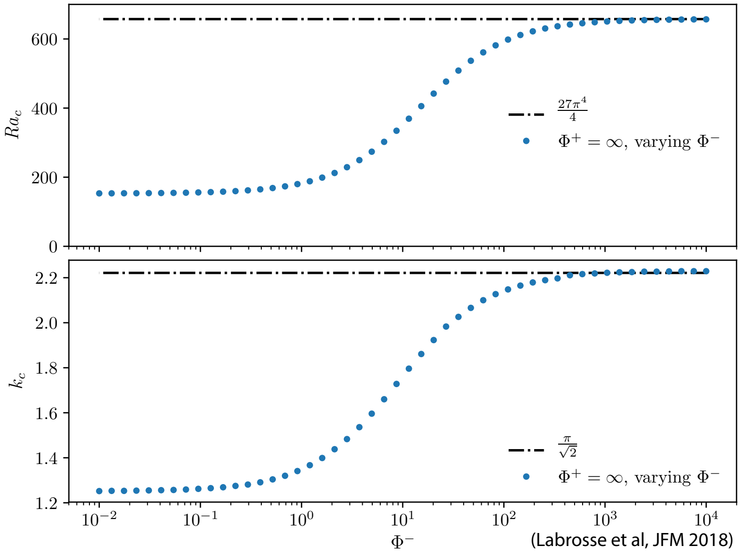
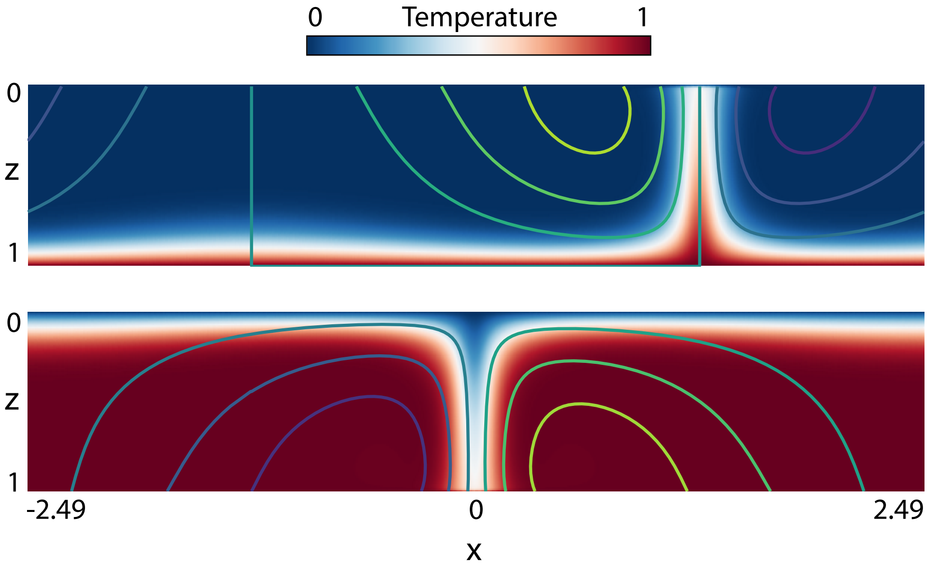
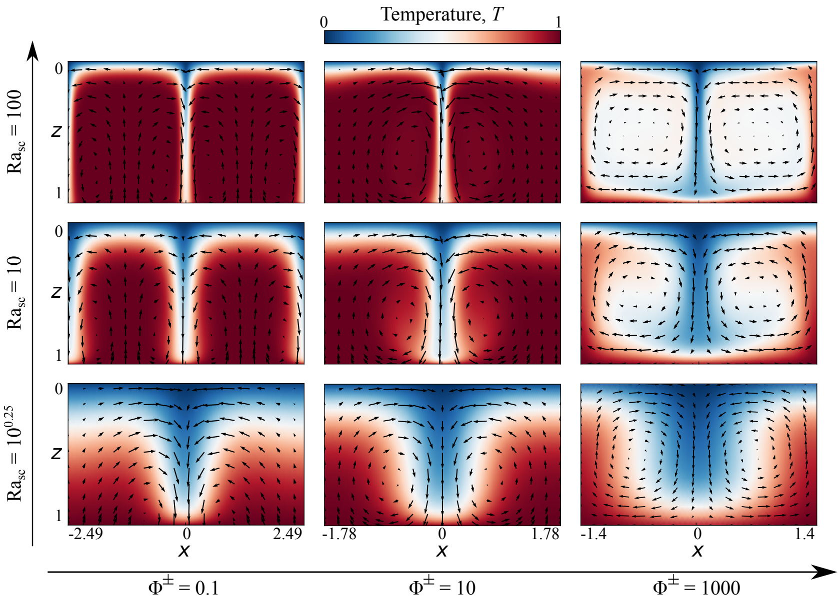


(Labrosse et al, JFM 2018)

Ra_c decreased by a factor ~ 4, k_c decreased by a factor ~ 2

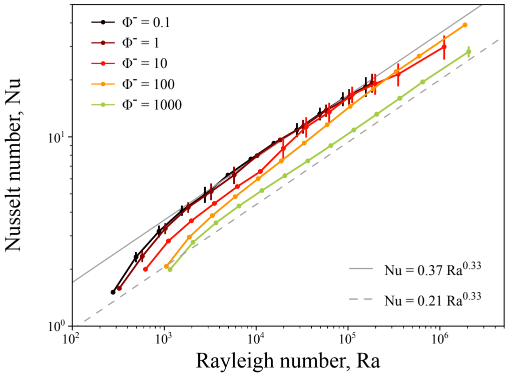
Thermal structure with one boundary with $\Phi = 0.1$

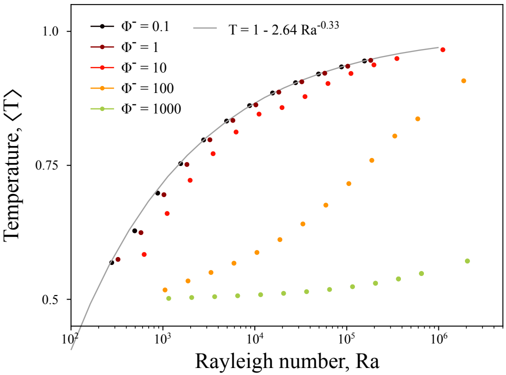




Temperature, T
0
1
$\mathrm{Ra}_{sc} = 100$
$\mathrm{Ra}_{sc} = 10$
$\mathrm{Ra}_{sc} = 10^{0.25}$
z
0
1
-2.49
0
2.49
-1.78
1.78
-1.4
1.4
x
$\Phi^{\pm} = 0.1$
$\Phi^{\pm} = 10$
$\Phi^{\pm} = 1000$

Heat transfer and mean temperature - high Rayleigh number





- At high Ra, $Nu \sim C Ra^{1/3}$.
- Coefficient C larger for small $\Phi \Rightarrow$ heat flow about twice larger for a given Ra.
- Consistent with a dynamics controlled by the only active boundary layer.

Outline

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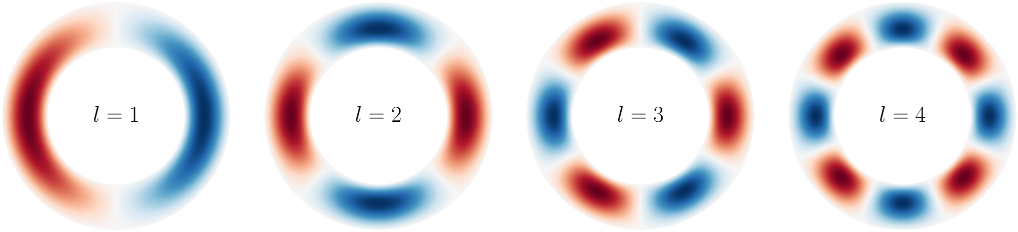
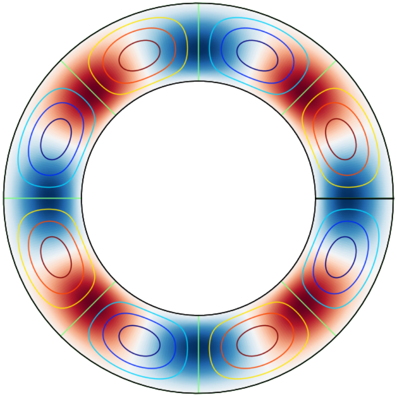
Spherical shell geometry

Morison et al, submitted to *Geophys. J. Int.*

- An additional parameter: the aspect ratio $\gamma = R^- / R^+$
- Linear stability analysis.
- Application to the onset of convection during magma ocean crystallisation (Morison et al., 2019).
- Direct numerical simulations.

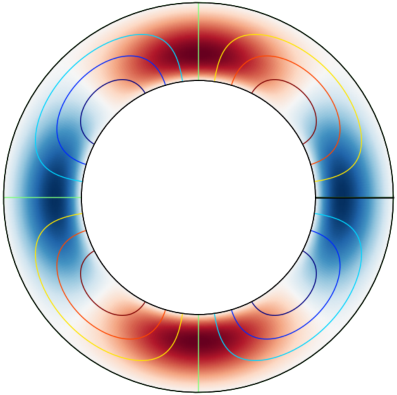




$$\Phi^{+} = 10^{4}$$
$$\Phi^{-} = 10^{4}$$

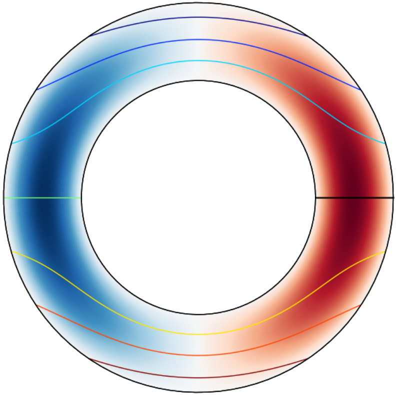
- $Ra_c = 687$ and $l_c = 4$
- Roughly square rolls
- Similar to classic non-permeable case

Linear Stability – Results



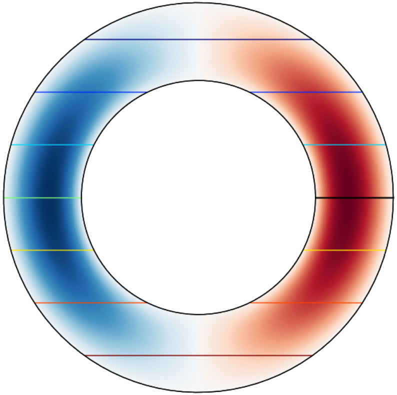
$$\Phi^+ = 10^4$$
$$\Phi^- = 10^{-2}$$

- $Ra_c = 188$ and $l_c = 2$
- Flow-through at the bottom
  - Half cells
  - Twice as wide
- Return current in the liquid ocean



$$\Phi^+ = 10^{-2}$$
$$\Phi^- = 10^4$$

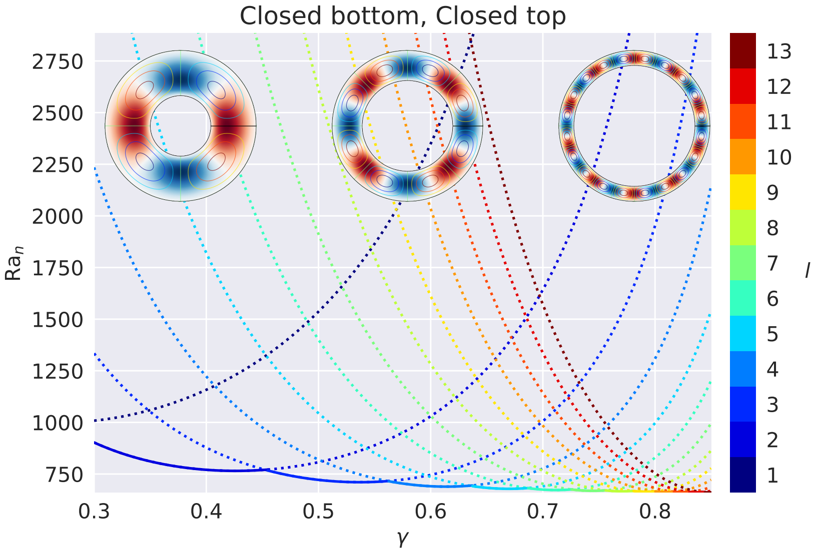
- $Ra_c = 96$ and $l_c = 1$
- Quasi-translation mode
- Very little deformation in the solid



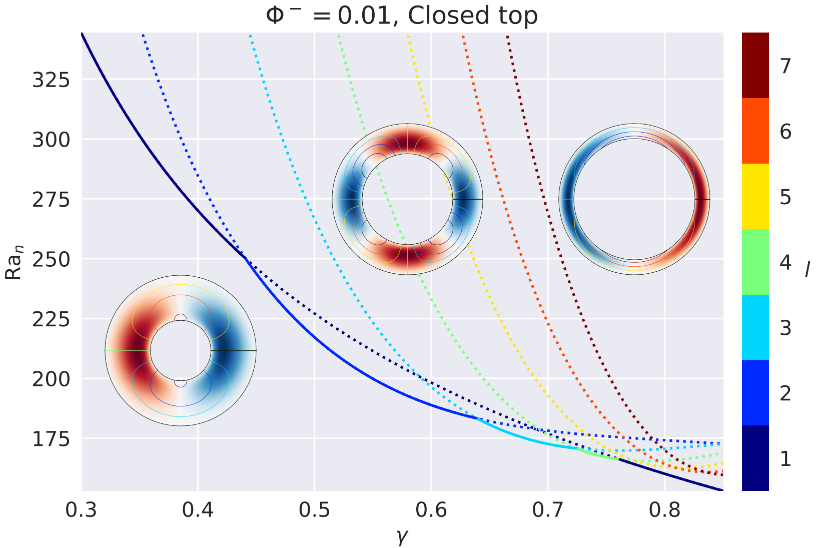
$$\Phi^+ = 10^{-2}$$
$$\Phi^- = 10^{-2}$$

- $Ra_c = 0.11$ and $l_c = 1$
- Translation mode without deformation
- Only limited by phase change

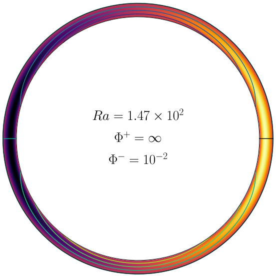
Effect of γ – Classical case

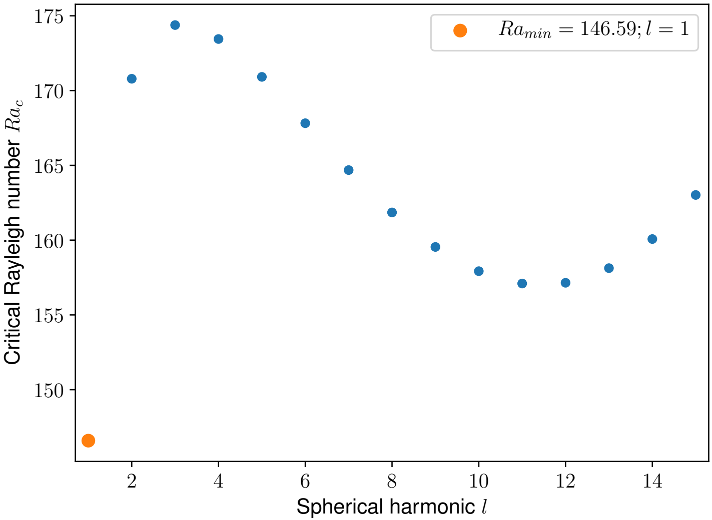


Effect of γ – Open at the bottom

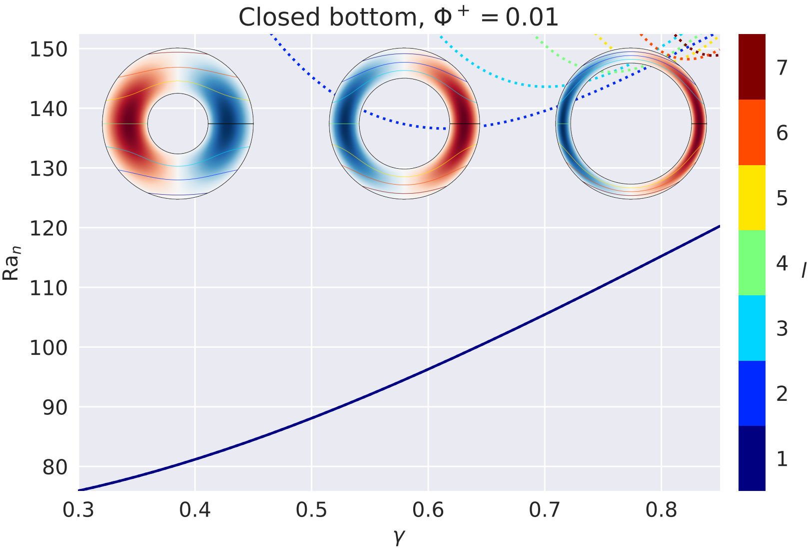


Competition between modes

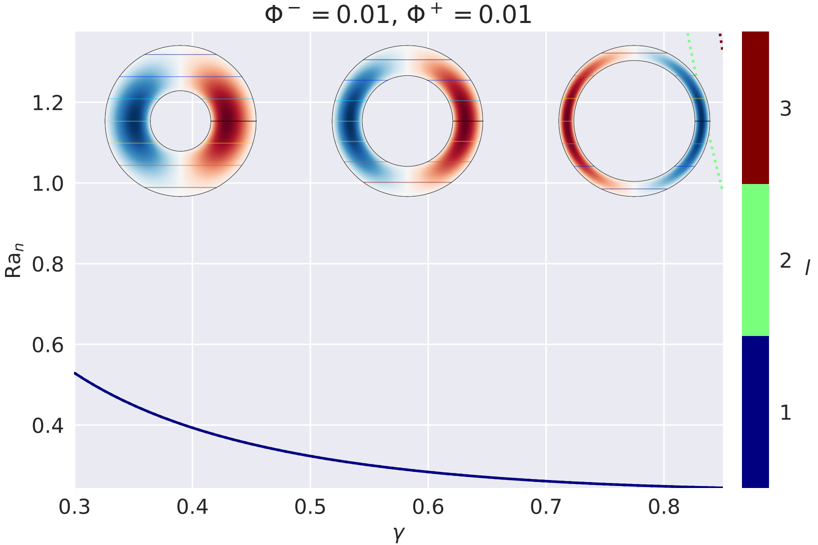




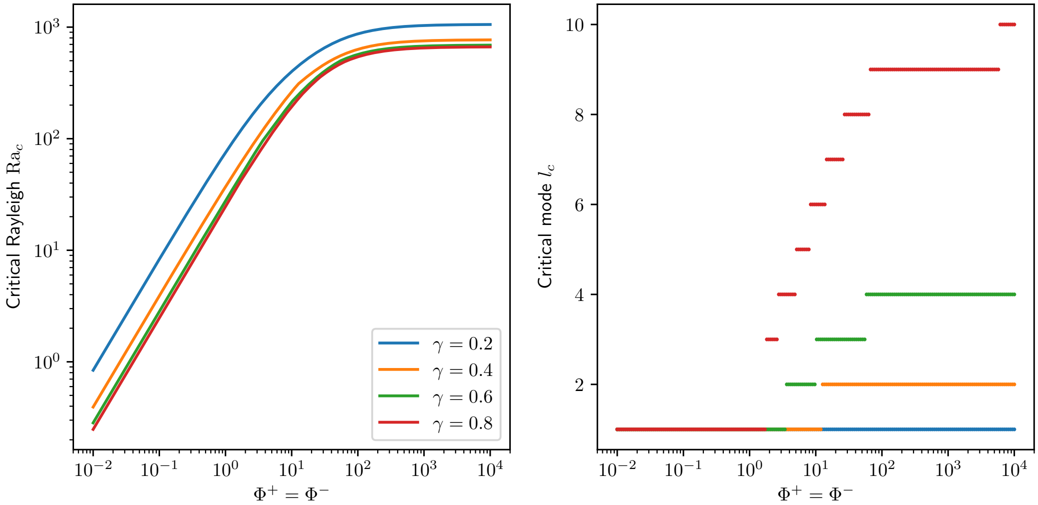
Effect of γ – Open at the top



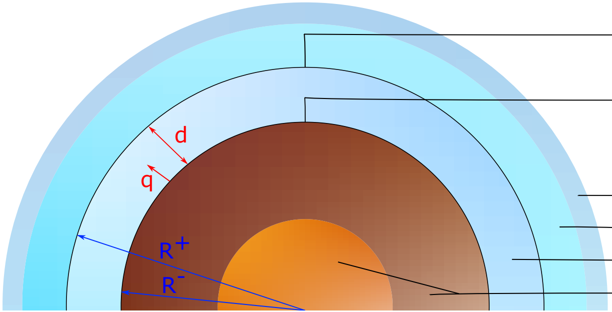
Effect of γ – Open at both boundaries



Exploration of parameter space $(\gamma, \Phi^{\pm})$



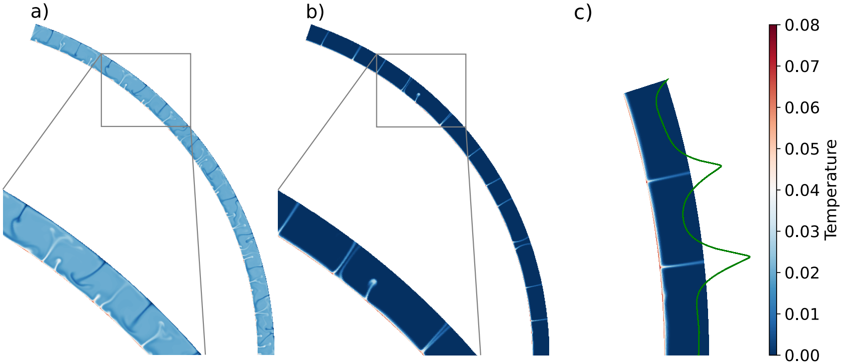
Convection in high pressure ice layers
(Lebec et al., 2024, 2023)



Heat and salts transfer between the rocky core and the ocean? A key question for the habitability.

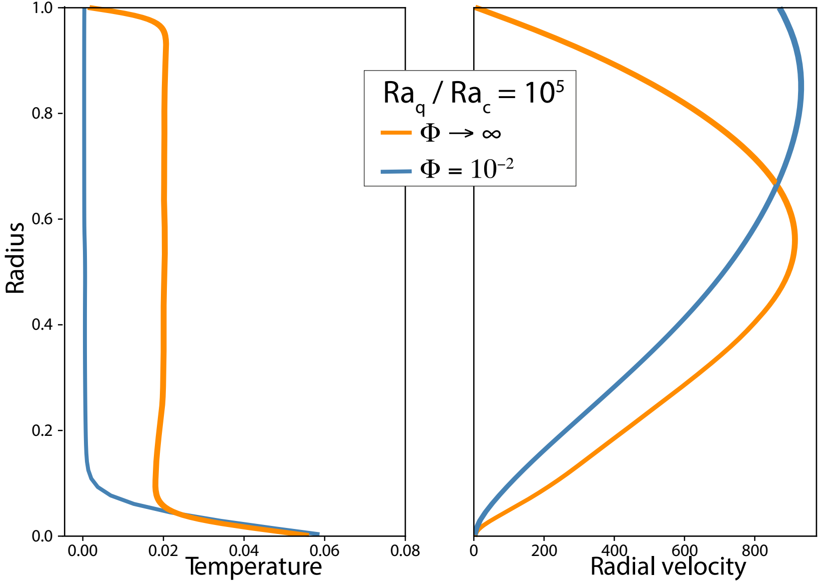
- How efficient is thermal convection in the HP ice layers?
- Do salts strongly influence the convective dynamics?

Thermal convection models

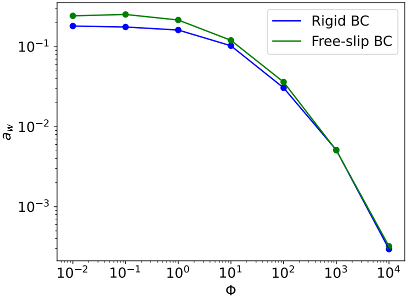


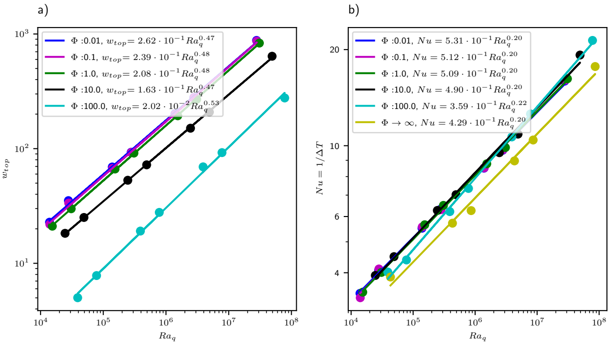
- Large radii ratio models: $\gamma = 0.9; 0.95$
- From slightly supercritical to large Rayleigh number (up to $Ra = 10^8$).
- Value of phase change number explored systematically.

Temperature and radial velocity profiles



Scaling for $\gamma = 0.95$





- Modest effect on heat transfer but large effect on mass transfer across the top boundary.
- $\Phi \lesssim 1$ already in the low-Φ asymptotic regime.
- Exponents well understood with classical boundary layer model.

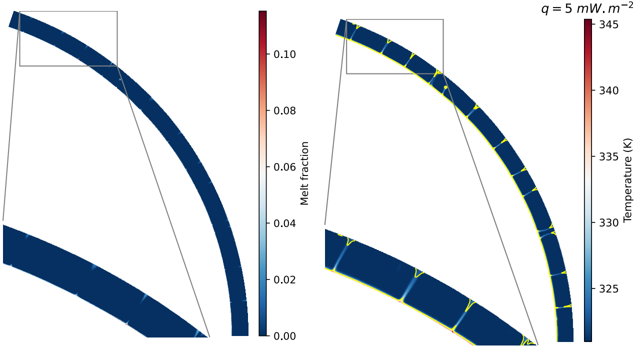
Keeping the least constrained parameters apparent:

$$Ra_q = \frac{\alpha g q \rho d^4}{k\kappa\eta} = 4.85 \times 10^8 \left(\frac{q}{10\,\mathrm{mW\,m^{-2}}}\right)\left(\frac{d}{100\,\mathrm{km}}\right)^4\left(\frac{\eta}{10^{15}\,\mathrm{Pa\,s}}\right)^{-1}$$

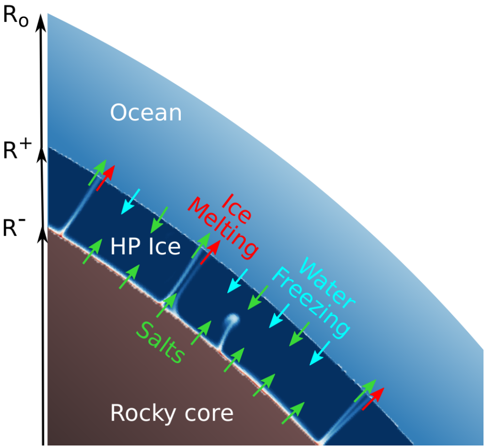
$$w_{top} = 0.262 Ra_q^{0.47}\frac{\kappa}{d} = 42.9\,\mathrm{cm\,yr^{-1}}\left(\frac{q}{10\,\mathrm{mW\,m^{-2}}}\right)^{0.47}\left(\frac{d}{100\,\mathrm{km}}\right)^{0.88}\left(\frac{\eta}{10^{15}\,\mathrm{Pa\,s}}\right)^{-0.47}$$

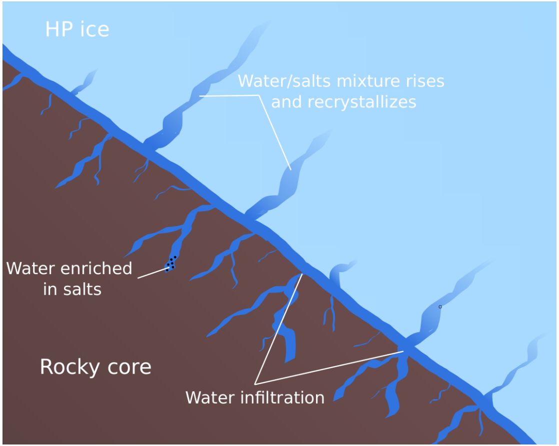
$$\frac{1}{\overline{\Delta T}} = 0.531 Ra_q^{0.2}\frac{k}{qd} = 4.6\times 10^{-2}\,\mathrm{K^{-1}}\left(\frac{q}{10\,\mathrm{mW\,m^{-2}}}\right)^{-0.8}\left(\frac{d}{100\,\mathrm{km}}\right)^{-0.2}\left(\frac{\eta}{10^{15}\,\mathrm{Pa\,s}}\right)^{-0.2}$$

- For most parameters choices, bottom temperature is close to melting temperature of HP ice.
- Computation of the position of melting line and melt fraction that should be produced.



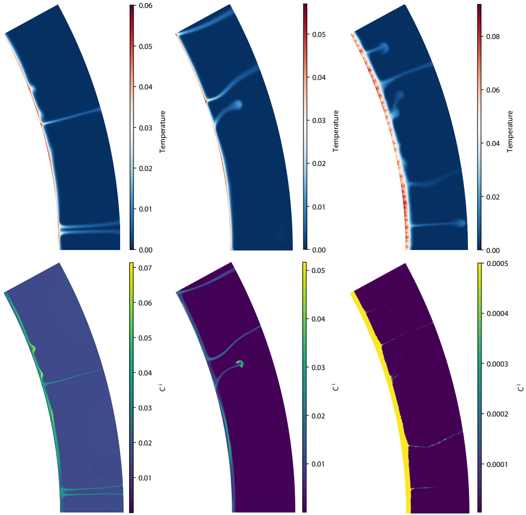
Effect of salts
(Lebec et al., 2024)





- Liquid water in contact with the rocky core ⇒ enrichment in “salts”.
- Salty water penetrates in the ice layer and freezes.
- Effects on convection in the ice layer?

Effect of salts - results



- Additional parameters: Buoyancy number B_{salts} and partition coefficient K.
- For $B_{salts} \lesssim 0.4$, no strong effect of salts on thermal convection.
- For $B_{salts} \gtrsim 0.6$, development of stratification.
- limited untrainment of salts to the upper layer such that effective B is small $\Rightarrow$ heat and mass transfer in the upper salt-poor layer similar to cases without salts.
- Overall, efficient transfer of salts toward the ocean.

Conclusions and outlook

- Convection in the solid is greatly influenced by the possibility of melting and freezing at either boundary.
- When both boundaries have a phase change, a translation mode becomes possible and accessible at very small Rayleigh number.
- Heat and mass tranfer is improved when a phase change boundary condition is considered.
- Application to the HP ice layers of large ocean worlds shows that heat and solute transfer between the rocky core and the ocean can be efficient.

bonuses

References I

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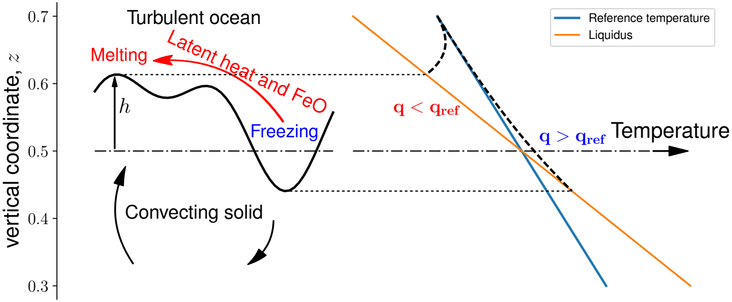
Morison, A., S. Labrosse, R. Deguen, and T. Alboussière (2019). Timescale of overturn in a magma ocean cumulate. *Earth Planet. Sci. Lett.* **516**: 25–36.

Phase change boundary conditions - 1

First developed for the inner core (Deguen, Alboussière, Cardin, et al)



- At the boundary: continuity of the temperature:

$$T(h) = T_m(h),$$

- At the fixed computation boundary, this leads to

$$T\left(\frac{1}{2}\right) = T + \left(\frac{\partial T_m}{\partial z} - \frac{\partial T_0}{\partial z}\right) h \Rightarrow \theta\left(\frac{1}{2}\right) = \left(1 + \frac{d}{\Delta T}\frac{\partial T_m}{\partial z}\right)\frac{h}{d}.$$

- Small topography: $\theta(1/2) = 0$

Phase change boundary condition - 2

- Energy conservation across the boundary, with v_ϕ the freezing rate:

$$\rho_s L v_\phi = [\![q]\!].$$

- Assume the convective heat flow on low–viscosity liquid side, $f \sim \rho_l c_{pl} u_l \delta T_l$, dominates. Temperature variations are associated with topography so that:

$$f \sim -\rho_l c_{pl} u_l \left| \frac{\partial T_m}{\partial z} \right| h.$$

- This gives

$$\rho_s L v_\phi \sim -\rho_l c_{pl} u_l \left| \frac{\partial T_m}{\partial z} \right| h \Rightarrow v_\phi = \frac{h}{\tau_\phi}$$

with τ_ϕ the phase change time scale hence defined.

Phase change boundary condition - 3

- Continuity of the vertical stress:

$$-p + 2\eta \frac{\partial w}{\partial z} + \Delta\rho g h = 0.$$

- Taking U as scale for the convective flow in the solid, this provides a scaling for the topography, $h \sim \eta U/\Delta\rho g d$ or $h = h'\eta U/\Delta\rho g d$.
- The topography evolves by phase change and viscous stress in the solid:

$$\frac{\partial h}{\partial t} = u_z + \frac{h}{\tau_\phi}$$

- Considering τ_c the time scale for the change of convective flow, using U as velocity scale, this equation is made dimensionless, with $\tau_\eta = \eta/\Delta\rho g d$:

$$\frac{\eta U}{\Delta\rho g d}\frac{1}{\tau_c}\frac{\partial h'}{\partial t'} = U u_z' + \frac{\eta U}{\Delta\rho g d}\frac{h'}{\tau_\phi} \Rightarrow \frac{\tau_\eta}{\tau_c}\frac{\partial h'}{\partial t'} = u_z' + \frac{\tau_\eta}{\tau_\phi}h'$$

- The time scale for the change of convective flow, $\tau_c \gg \tau_\eta, \tau_\phi$ and we can neglect the left–hand–side. In dimensional form, $u_z = -h/\tau_\phi$. Used in the stress–continuity equation to eliminate h.

Phase change boundary conditions - 4

- The same can be done for the bottom boundary condition. Beware: the sign of $\Delta\rho$ is reversed.
- Dimensionless boundary condition for vertical velocity:

$$\pm\Phi^{\pm} w + 2\,\frac{\partial w}{\partial z} - p = 0, \qquad \text{with} \quad \Phi^{\pm} = \frac{\tau_\phi}{\tau_\eta} = \frac{\tau_{\phi^\pm}|\Delta\rho^{\pm}|gH}{\eta}$$

- $\Phi \rightarrow \infty \Rightarrow$ classical non-penetrative boundary condition ($w = 0$).
- $\Phi \rightarrow 0 \Rightarrow$ permeable boundary condition ($w \neq 0$).
- This boundary condition expresses the competition between the building of topography from stress in the solid and its suppression by convection in the liquid.

Linear stability for deforming modes

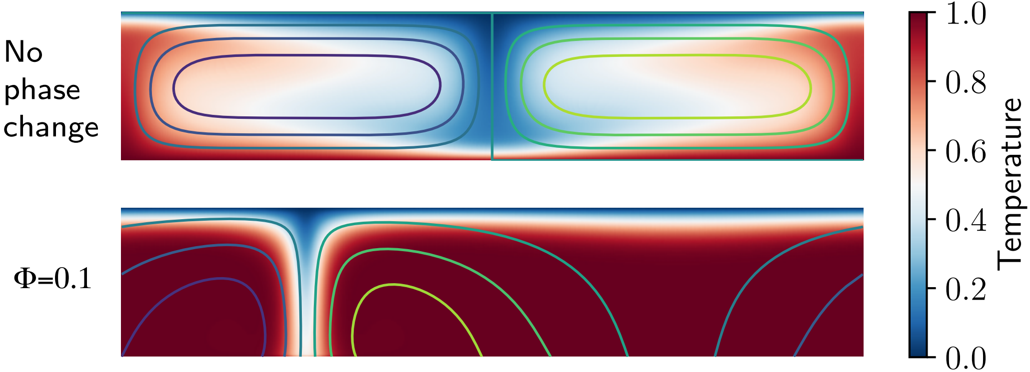
Find the critical Rayleigh number and the associated flow for the onset of convection as function of Φ^+ and Φ^-:

- The conservation equations for mass, momentum and temperature are linearly developed around the motionless conductive solution.
- A simple harmonic in horizontal direction:

  $$\theta(x,z) = \Theta(z)\mathrm{e}^{\sigma t + \mathrm{i}kx} + c.c.; \quad w(x,z) = W(z)\mathrm{e}^{\sigma t + \mathrm{i}kx} + c.c.; \quad \text{etc.}$$

  with the wavelength $\lambda = 2\pi/k$.
- For each k, we search for the Rayleigh number $Ra(k)$ which makes $\Re(\sigma) = 0$ (neutral stability).
- The minimum of $Ra(k)$ gives the critical Rayleigh number Ra_c and the associated wavenumber k_c.
- Full calculation performed using a Chebyshev-colocation method, behaviour for small $\Phi^\pm$ obtained analytically by polynomial expansion in z and $\Phi^\pm$.

Thermal convection with melting/freezing at the bottom



- Critical Rayleigh number $\sim 4\times$ smaller and critical wavelength $\sim 2\times$ larger than classical (Labrosse et al, JFM 2018).
- Finite amplitude solution:
  - Only down-welling currents are focused (like for internally heated convection).
  - Heat transfert much more efficient than with classical convection (Agrusta, et al, 2019).